A Theoretical Foundation for Technical Analysis[1]

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1

Abstract

Using a dynamical microeconomic model which generalizes the classical theory of adjustment to include finite asset base and trend-based investment preference, we develop a foundation for the technical analyis (or charting) of securities. The mathematically complete system of (deterministic) ordinary differential equations that has provided a quantitative explanation of the laboratory bubbles experiments generates a broad spectrum of patterns that are used by practitioners of technical analysis. The origins of many of these patterns are classified as (i) those that can be generated by the activities of a single group, and (ii) those that can be generated by the presence of two or more groups with asymmetric information. Examples of (i) include the head and shoulders, double tops, rising wedge while (ii) includes pennants and symmetric triangles. The system of differential equations is easily generalized to stochastic ODE'S. Application is also made to Japanese candlestick analysis.

1. Introduction

Technical Analysis Methodology

Technical analysis or charting is a technique which uses the patterns of the price history of a financial instrument (commodity, currency, stock or composite average) in order to provide indications on the future behavior of prices [Edwards and Magee (1992), Myers (1989), Pring (1993)]. Technical analysis actually consists of numerous methods with a common set of basic principles, and is the chief alternative to fundamental analysis which strives to assess the true value of financial instruments. The philosophical basis of fundamental analysis is quite close to classical economic theory which stipulates that prices will move in a direction to alleviate the discrepancy between the current price and the true value. Thus, the investor tries to discover a stock, commodity or financial instrument whose current price does not yet reflect its improved fortunes. The investor who trades on such a fundamental basis thereby becomes part of the process by which prices are returned to equilibrium while profits are made in the process. Traders who prefer technical analysis usually agree that the fundamental value will eventually be attained. However, they contend that funds can remain unproductive for the intermediate term (from days to months) while awaiting the long term process of reaching the equilibrium or fundamental value which may take years. Furthermore, they contend that even the direction of prices need not be uniformly in the direction of fundamental value. The implicit assumption is that the dissemination of information and reassessment of value is a slow process which may be overshadowed by sellers of undervalued securities who are either unaware of true value or hesitate to rely on the optimizing behavior of others (see Section 3) and are attempting to limit losses. The immediate direction of prices, they argue, is determined solely by the supply and demand of the financial instrument, and that the changes in supply and demand are generally evident upon examining recent price trends.

Unlike fundamental analysis, technical analysis has no basis in classical economic theory. For example, a decline in prices for an undervalued stock can only be attributed to random fluctuations in the context of basic theory. Thus, the widespread practical use of methods of technical analysis which are contradictory to classical economic theories provides three important puzzles for mathematical economics. First, can one explain the key patterns of technical analysis based on a generalization of the classical theory of adjustment? Second, can one verify using statistical hypothesis testing that such patterns actually appear in real markets? Third, do such patterns have a predictive value within the context of a model or in real markets?

In this paper we focus attention mainly on the first issue and partly on the third by using a model which generalizes the classical theory of adjustment to include supply and demand changes which depend on price derivative in addition to price [Caginalp and Balenovich (1991), (1994b)]. Within this approach, the classical (value-based) motivation for purchasing the equity is augmented with a trend-based strategy of buying as a result of rising prices. The analysis of supply, demand and price, based on money flow, as a function of time leads to a system of differential equations, which may be deterministic or stochastic. The model incorporates, in a natural way, the conservation of asset principles which prevent a bubble from continuing indefinitely, and includes the effects of finite markets that are absent in the classical theory. These equations have been useful in understanding the non-classical behavior of real and experimental markets such as the formation of bubbles and subsequent fall in prices. In particular, the parameters in the equations can be calibrated with a single experiment such as those of Porter and Smith (1989), (1994); the remaining experiments are then predicted with no adjustable parameters. Statistical tests confirm that the predictions are overwhelmingly more likely than the alternative hypothesis, either in the form of the efficient market hypothesis or in the trivialization of the trend based component.

The use of technical analysis has always posed an interesting question for the efficient market hypothesis as the latter implies that such methods could not be successful. In particular, the weak form of the efficient market hypothesis maintains that prices incorporate all public information so that an analysis of price pattern cannot produce any profits. Moreover, significant return from technical analysis, even in conjunction with valuation methods, tends to argue against the efficient market hypothesis. Consequently, there is a close link between the validity of technical analysis and the inefficiency of the market. For example, the bubbles that have been observed in laboratory experiments are an example of inefficiency. At the same time, it would be easy to use momentum indicators, for example, to profit from these bubbles if one were participating in these experiments. Also, the necessity for technical analysis in securities marketing, e.g., secondary offerings, becomes evident if the relevant market is not very efficient. We discuss some recent examples of market inefficiency below.

The second question of statistical testing based upon patterns is not addressed in this paper, though it has been studied in others. Brock et al. (1992) review the literature and conclude that many simple rules have found no statistical validity. In particular, they test the hypothesis of the moving average trading rule that consists of a buy signal when the price moves above a particular moving average and a sell signal when the price crosses below the average. Using the data set of the Dow Jones average for several decades, they found almost no net gain for using either the buy or the sell signal. White (1993) analyzed neural networks on 1000 closing prices of IBM stock that was used to make predictions on the next 500. Roughly speaking, White's procedures involved simply finding the optimal fit for the past three days of closing prices and utilizing it to predict the next day. The procedure failed to produce a profitable trading strategy. To technical analysts these failures are not surprising. An examination of simply closing prices during three days is unlikely to capture the emotion of the previous days. Also a relatively small sample such as 500 data points for one stock will not have the capability to uncover particular patterns.

However, a recent study by Blume et al. (1994) explores technical analysis as a component of agents' learning process. Focusing mainly on the informational role of volume, they conclude that sequences of volume and price can be informative and argue that traders who use information contained in the market statistics attain a competitive advantage.

A different approach, adopted by Caginalp and Laurent (1998) involves testing of short term patterns, called Japanese Candlesticks, believed to have predictive power. Scientifically testing all three day reversal patterns discussed in Morris [1992], on a 265,000 day data set of daily open, close, high, low for each of the S&P 500 stocks for a five year period, they found significant predictive power. In other words purchasing or selling short at the end of a three day candlestick pattern (with the correct trend properties as defined in this paper) and selling (or buying back in the case of short positions) one-third of the shares on each of the next three days leads to statistically significant profit. Even the short sales during this rising market (1992 to 1996) lead to average profits of about 0.27% for an average of two trading days. The three day reversal patterns (see Section 5) they considered have basic explanations in terms of the psychology of trading. We discuss this in terms of our differential equations model.

In recent years several authors have revisited the tests of classical technical rules (e.g. moving average) under slightly different conditions and have often concluded that a slight advantage, usually below trading cost, is attained. These include Antoniou et al.. (1997) who find that price trend integrated with volume yield some predictability in the emerging market of Istanbul.

Similarly, Bessembinder and Chan (1998) included dividends in the returns and found some positive return. Chang and Osler (1999) found that the head and shoulders pattern, defined below, was predictive in some cases and not in others. Chan et al. (2000) found that momentum strategies (particularly if augmented by volume considerations) have some significant positive returns for international stock indexes for holding periods less than four weeks. Other works such as Maillet and Michel (2000) examine technical analysis with filtering methods on foreign exchange markets, and conclude that these methods can be used to improve upon naïve rules.

Other works have sought to define and identify computational patterns such as triangle patterns, etc. (Kamijo and Tanigawa [(1993) and Lo et al. (2000)]. Statistical testing of computationally well defined patterns remains largely a current research topic.

Nonclassical Phenomena in Equities Markets

The classical theories of financial markets that have largely dominated the academic studies of financial markets are generally based on the concept of random price fluctuations about a fundamental value [e.g., Fama (1970), Tirole (1982)]. A theoretical basis for this idea is that any deviation from a realistic value is reason for investors to purchase (due to undervaluation) or sell short (due to overvaluation) in the expectation that a profit can be made as other investors follow suit. The following three prerequisites appear to be part of any microeconomic model of this concept: (i) There are a group of investors with similar assessments of the value of the security. (ii) These investors have capital that exceeds those of less informed investors. (Or, alternatively they manage the capital of less informed investors who have learned of their investing skill through a perfectly accurate representation of managers' likely performance.) (iii) These investors not only want to maximize their returns but are also willing to rely on the self-interested maximization of others.

It will be evident in some of the examples discussed below that (iii) is a crucial component in that the deviation from true value typically does not in itself guarantee a profit in an equity, though it may in Treasury bonds that have an expiration that is within the investor's time horizon, for example. An overvalued security can become more overvalued and the short position must be covered before it exceeds margin requirement. The investor who shorts 1000 shares of a $1 stock is forced to buy back those shares (and contribute to further price elevation) if the stock subsequently trades at $2, or put up additional cash (under current rules). Consequently, taking such a short position involves relying upon other investors to take similar actions in self-interested maximization.

Although it would appear that investors would implement such a strategy, the laboratory experiments of Beard and Beil (1994) — to be discussed in Section 3 — indicate a reluctance of players to rely on the self-interested maximization of others even while they engage in self-interested optimization themselves, though they become more willing to do so when the cost of not maximizing becomes very high for the other player.

The variation in the degree of validity conditions (i) - (iii) in different markets leads to the question of the extent to which markets are near fundamental value, or semi-strong efficient, meaning that public information cannot lead to profitable trading.

As with most idealizations, the efficient market hypothesis (in each of its forms) varies in the extent of its applicability depending on the validity of the underlying assumptions. In the Treasury bill market, for example, conditions (i) and (ii) appear to be valid due to the huge size of the market, the broad participation of financial institutions and the clear nature of the fundamental value. The validity of (iii) can be expected in this market since a player who buys a six month Treasury bill, even though he may need the money in one month, is relying on the self-interested maximization of other players who would be sacrificing a large and certain profit by not purchasing a discounted bill with five months left to maturity. At the opposite extreme would be the low capitalization stocks, emerging markets, etc., where bubbles and other deviations from fundamental value seem to occur most frequently. The low level market capitalization and trading volume tends to deter larger investors since the total potential profit, which may be large by percentage measurements, is small in comparison with the investor's assets, and, moreover, the profits may be very difficult to realize due to the limited liquidity. Relying upon the self-interested maximization of others appears to be most difficult when there is much uncertainty about their potential profits and even about the existence of such players!

In a study examining the evolution of major US stocks during the past century, Shiller (1981) concludes that volatility over the past century appears to be five to thirteen times too high to be attributed to new information about future real dividends. Understanding market efficiency or ineffiency is often difficult because 'noise makes it very difficult to test either practical or academic theories about the way that financial or markets work,' according to Black (1986). He also states that markets are efficient 90% of the time if one takes a reasonable definition of efficiency such as one in which 'the price is more than half of value and less than twice value.' This allows for a quadrupling in price and is well worth investigating from both a theoretical and practical perspective. Our approach is based on the idea that there is a deterministic asset flow based on trend and value, in between the efficient valuation ideas that give a coarse idea of price and the fine grain fluctuations that are due to pure randomness.

We present below some recent examples of market phenomena that are not explained by the classical theories unless they are augmented with the concepts of a trend based aspect of preference and the flow of a finite supply of assets.

i) Many, though not all, common stocks of bankrupt companies are worthless, since the common shareholders are legally the last in a long line of creditors for whom there is insufficient funds. Usually even the price of the senior debt reflects a market expectation that the bond holders will receive almost nothing while the common stock, which will receive much less, continues to trade as though it had significant value. In some cases, e.g. Continental Air in 1992, the common stock had already been pronounced worthless by a bankruptcy court judge in accepting a reorganization plan. Nevertheless, the stock continued to trade and rally for weeks after the decision. This particular example illustrates some of the difficulty in testing efficient market theories. The usual test is that if the market is inefficient, then there should be a method for profiting from it. Yet if we consider a worthless stock that rallies from ten cents to two dollars and eventually drops to zero, it is clearly a (nearly) zero-sum game: some players win, some lose and probably all have some sort of strategy. Even among the players who are quite certain that the stock is worthless, there will be losers among those less skilled in anticipating the actions of the majority of investors. The lack of a precise algorithmic method that always can profit from such a situation certainly does not change the obvious conclusion that this is an inefficiency (see also Section 5). The argument that such a stock must have some value, since one can reasonably expect to sell it to someone else at a higher price, reduces the efficient valuation theory to a tautology.

ii) In some cases, large capitalization stocks that have been privatized by a government have risen to a market valuation that cannot be explained on a fundamental basis. For example, in 1989, as the Japanese government privatized the phone company, Nippon Telephone and Telegraph Co. (NTT) had a market value that exceeded the entire West German market. The theories of efficient markets are not only confronted with the puzzle of valuation but of the possible influence of the sale process on the price.

iii) Many of the world's markets have suffered severe drops that appear to exceed the change in fundamental value after long trends that exaggerated value. The huge boom and subsequent crash in the Internet/high-tech part of the US market is a reminder that bubbles can occur in markets with rapid information flow and high liquidity. During the height of the bubble, many companies soared to market caps of billions without ever recording a profit. A large number of such stocks fell below 1% of their peak prices, as nearly seven trillion in market capitalization was obliterated by 2002. The NASDAQ lost approximately 80% of its value in the two year bear market that began in the year 2000. In the Japanese market, many other Japanese stocks besides NTT soared to excess valuation by the end of 1989, and the Nikkei lost half of its value in eight months in 1990. Similarly Taiwan had seen the price/earnings ratios of many companies reach triple digits until the Weighted Index lost over 80% of its value in eight months.

iv) Stock market crashes such as those of 1987, 1929 and the mini-crash of 1989 and subsequent rebounds appear to be inexplicable from a valuation perspective [see e.g. Kampuis (1989)] as is the large sell off in derivative and mortgage-backed securities in early 1994. The 1929, 1987 and derivative crash of 1994 may all be viewed as an exaggerated market response prompted by the Federal Reserve's rate increase and aggravated by the increased supply in the midst of declining demand for the securities.

v) An equally puzzling phenomena among closed-end funds is the large premium (50% to 100%) that has occurred particularly among country funds which generated a high degree of investor interest.

vi) The persistent discount from net asset value that afflicts the average closed-end fund has been a puzzle for both practical and academic observers, since investors with cash can attempt to maximize their returns by buying $100 of stock for $80 in many cases. During 2001 some of the discounts exceeded 30% so that $66 could buy $100 of stock in the case of India Growth Fund, for example. Some articles have debated whether the origin of this discount is related to investor sentiment [Anderson and Born (1992), Lee et al. (1991), Chen et al. (1993), Chopra et al. (1993)].

Secondary offerings of stock in major companies and closed end funds have resulted in lowering prices according to reliable statistical studies on Swiss stocks [Loderer and Zimmermann 1985] and common experience on Wall Street. Underwriting specialists generally acknowledge that additional supply of a closed-end fund or common stock will "weigh" on the market and attempt to estimate the amount of new supply that the market will bear. However, the efficient market hypothesis predicts that other investors would eliminate any discount by purchasing the shares and cannot address this issue. The removal of stock has essentially the opposite effect. Clearly, the efficient market is contingent upon a large supply of additional funds in these situations.

The spectacle surrounding initial public offerings (IPO's) reached a fever pitch during the late 1990s. Many newly formed companies soared to market caps of billions on their first day of trading, often with no sales, let alone profits [Loughran (2002)]. Even the most generous fundamental assessment led to the conclusion that these shares were worth pennies rather than $20 per share or higher. When a stock is trading for one hundred times its fundamental value it is difficult to attribute the price changes to fundamentals or even new information. The price changes are most likely influenced by traders' perceptions of others' motivations.

These real world market examples are, of course, complemented by the controlled laboratory experiments such as those of Smith (1982), Plott and Agha (1982), Forsythe et al. (1982), Porter and Smith (1989), (1994); Smith et al. (1988a,b); Williams and Smith (1984) [see also Porter and Smith (1994) for other references] in which prices overshoot the fundamental value and subsequently crash.

An Asset Flow Perspective in Securities Marketing and Technical Analysis

We will consider these examples in the context of our asset flow differential equations [see Caginalp and Balenovich (1991), (1994b) and references therein] which incorporate the ideas of a finite asset base, a trend based component of investor preference, and the possibility of distinct groups with asymmetric investors (Section 3). The model can naturally incorporate the flow of additional stock or cash and can be generalized to include a variety of investor strategies and motivations. Moreover, once the parameters in the system of equations are calibrated for an investor population using a single experiment, the equations can then be utilized to obtain quantitative predictions for any hypothetical scenario or set of quantitative assumptions such as the magnitude of assets of distinct groups. The classical theories offer a limited amount of insight into the problems that confront practical securities marketing in that the theories tend to assume infinite arbitrage capital and infinite rationality. Our analysis is designed to provide a basic framework in which the flow of various components of supply and demand is tracked explicitly within the context of preferences that depend on valuation and the price trend.

In addition to the examples above there is a large body of knowledge known as technical analysis which attempts to identify the patterns on price charts that may offer an indication of whether a trend is likely to continue or terminate. Of course, such a possibility is ruled out by the (weak) efficient market hypothesis which maintains that prices alone have no predictive value, and so many academicians are quite skeptical of these ideas, while practitioners use them routinely in trading and marketing securities. We will discuss some of the major patterns in Section 3 and then describe how these patterns are a natural consequence of this asset flow model, thereby providing a theoretical basis for technical analysis.

2. Technical Analysis

Basic Assumptions and Introduction

In their lead sentence, Caginalp and Laurent [1998] observed that "The gulf between academicians and practitioners could hardly be wider on the issue of the utility of technical analysis."[2] While economic and financial scholars often ignore or downplay the role of conventional technical analysis, financial experts are often quite eager to implement it without regard to the nature of the economic assumptions inherent in the methods. We present here a brief summary of some simple patterns and the basic principles that standard references state as the justification for technical analysis [Edwards and Magee (1992), Myers (1989), Pring (1993)]. The key assumptions are as follows:

1) Trends in prices tend to persist. This is essentially a momentum concept which means, in economic terms, that the supply/demand ratio is slowly varying (despite changing prices) unless there is a significant change in fundamentals or the sources of supply or demand. Note that this assumption may violate classical equilibrium economics in that a price rise is not expected to bring an immediate decline in demand or rise in supply. Thus, the validity of this key assumption of technical analysis appears to be contingent upon introducing price derivative dependence, in addition to price dependence, upon the demand function. This is an important issue from the perspective of microeconomic price theory as discussed in greater detail in Section 3, after equation (3.3).
2) Market action is repetitive. This assumption maintains that various patterns appear again and again in price charts. These patterns evolve as a consequence of investors' reactions to the change in their fortunes. Thus, the recurrence of various patterns is a manifestation of the tendency for people to behave similarly (or employ analogous strategies) in similar situations.

Rather than presenting the various price patterns in the traditional way (e.g., [Myers (1989)]), we organize these patterns in a manner that will be compatible with the asset flow theory, which provides a unified explanation for these phenomena. Most significantly, we divide patterns into two categories, one of which can be understood in terms of a single group of investors with similar information and motivations, and the other which requires at least two such groups. Furthermore, traditional explanations of charting tend to focus on the distinction between consolidation patterns (meaning the trend will resume after a brief respite) and reversal patterns (meaning the trend will terminate and be replaced by a trend in the opposite direction). While this distinction appears to be natural when the chief goal is to profit by predicting trends, our analysis shows that it is not very basic since the origin of some consolidation patterns is quite close to related reversal patterns. In fact, one of our conclusions is that it appears to be difficult to predict the direction of prices (using only price histories) before they emerge from a complex pattern such as a symmetric triangle as discussed below.

Our methods are aimed at showing that the major patterns of technical analysis can all be obtained from a set of assumptions that consist essentially of the classical theory of adjustment augmented with the concepts of (a) trend based investing, (b) finite asset base, and for some patterns, (c) asymmetric assessment of value. The conclusions we draw would not be significantly altered if a modifications of the asset flow model were used, provided that these ingredients are preserved and that the parameters are evaluated with the same principles (e.g. calibrated from experiments).

Evaluation and Criticisms of Technical Analysis

Criticisms of technical analysis have included the following:

a) The complex patterns are difficult to identify in an objective manner.

b) Academic studies [e.g. White (1993)] have indicated that positive out-of-sample results are not obtained in a straightforward manner.

c) There is a perception that pure technical analysts (who avoid valuation completely) do not produce consistently large profits.

The issue of objective identification has recently been considered within computer programs that eliminate personal biases [Kamijo and Tanigawa (1993)], though this endeavor is in its initial stages. The academic studies of price predictability have often focused on a particular algorithm (e.g. can one devise a neural network algorithm for using the past three days' closing prices to predict the fourth? [White (1993)]) that is far simpler than those used by practitioners, who are thus unpersuaded by these arguments. Finally, while it is difficult to ascertain the profitability of pure technical analysis, a simple argument illustrates the limitations involved in ignoring fundamental developments in valuation. A price pattern reflects the dynamics of supply and demand for the security up to the present time. As fundamental changes occur the pure technician relies on other traders to interpret the changes and make trades that change the pattern before the technician acts. Suppose we consider a pure technician in competition with an equally knowledgeable technician who is also adept at assessing changes in valuation. The latter will always have a substantial advantage in that he can open or close positions substantially before (and more favorably than) the pure technician. Moreover, an investor who is apprised only of fundamental value may not make the optimal trade if he is unaware of the changing balance of supply/demand. Thus, the technician/fundamentalist can be expected to outperform both the pure technician and fundamentalist. The latter two can be expected to outperform traders who have no information on value or supply/demand.

Single Group Patterns

Trendlines or Channels: One of the most common patterns or channels observed in financial patterns is the general movement of prices in either the upward or downward direction, roughly between two parallel lines as shown in Figure 1. The chief practical uses of trends are to identify them in order to buy securities in uptrends, trade within limits in horizontal channels and to determine when the trendline has been broken.

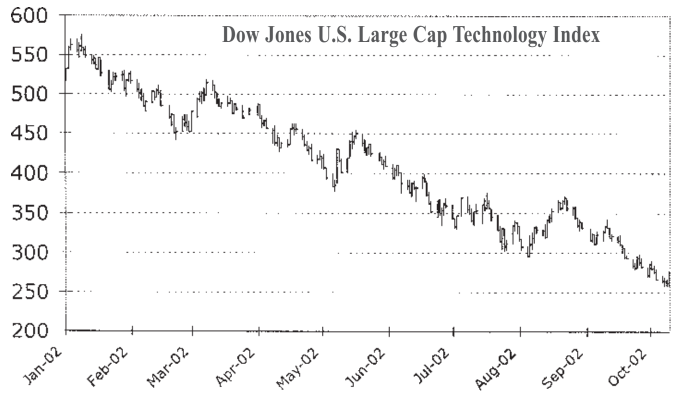


FIGURE 1: DOWNWARD TRENDLINE. The price chart for the large capitalization stocks during the year 2002 displays a nearly perfect *downward trendline* that began with the collapse of the overvaluation bubble in the US markets. The more significant part of the channel is the upper part of the trendline that provides the resistance. Technicians offer the advice that "the trend is your friend," or "don't fight the tape," meaning that the trend is likely to continue in the absence of a clear sign that indicates otherwise. This sometimes happens when the upper trendline is penetrated with a *gap*, or visible jump of prices across this line.

Gaps: In a strong uptrend, one often observes that a trading range for a day's prices lies entirely above the previous day's. The appearance of an upward gap is an indication of buying strength and higher prices for the immediate future. The situation is analogous for downtrends. If the latest gap is within the range of prices during a suitable period prior to the gap, then it is called insignificant.

Rounding Top or Bottom: A reversal pattern which is simple but not very common occurs when the price gradually changes direction.

Key Reversals, V-formations and Spikes: Another set of reversal patterns is similar to the rounding top except that they occur more abruptly. A V-formation bottom simply has the appearance of a V as is illustrated in Figure 2. A spike is a sharp V in which the maximum is attained and abandoned quickly. A key reversal is more specific in that a maximum is attained on a particular day which ends near the lowest part of the day's trading range.

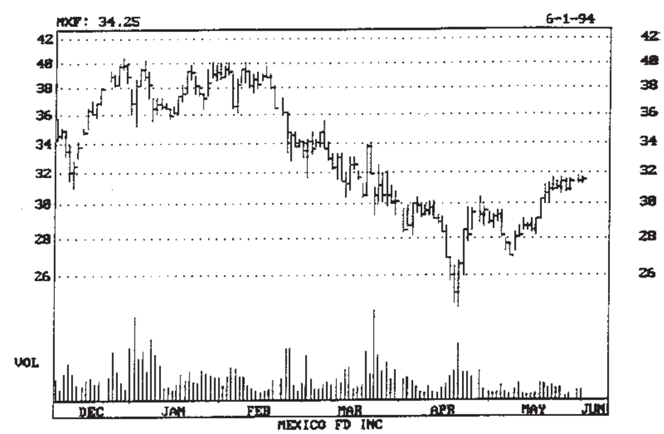


FIGURE 2: V-FORMATION REVERSAL: After a steady downtrend in the Mexico Fund, a sharp sell-off in April 1994 results in a *V-formation bottom* that is flanked by two local minima, the latter (in May) providing additional confirmation of the April support.

Head and Shoulders: A common reversal pattern with a more complex appearance, the head and shoulders pattern appears as a major peak flanked by a minor peak at either side. The analogous bottom is the inverted head and shoulders pattern shown in Figure 3. In some ways this is similar to the diamond formation in which the convex hull of the local maxima and minima form a diamond shape.

Double and Triple Tops (or Bottoms): A pattern which bears some resemblance to a head and shoulders top occurs when two or three peaks (with similar peak values) occur successively as they attain essentially the same maximum. Triple tops and bottoms are less common than double.

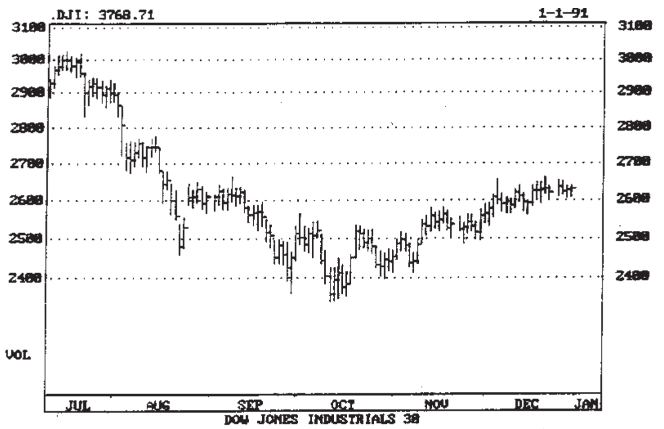


FIGURE 3: HEAD AND SHOULDERS. An *inverted head and shoulders pattern* appears in October 1990 in the Dow Jones Industrial Average (DJIA) as fears of an impending Gulf war and the possibility of contracted oil supplies and recessionary effects yields to increased optimism and a more accomodative monetary policy. Note also that the right shoulder, which is often slightly higher than the left, acts as a confirmation that the buying has come in at a higher price than either of the two local bottoms, thereby providing a strong confirmation of *support* above the 2400 level.

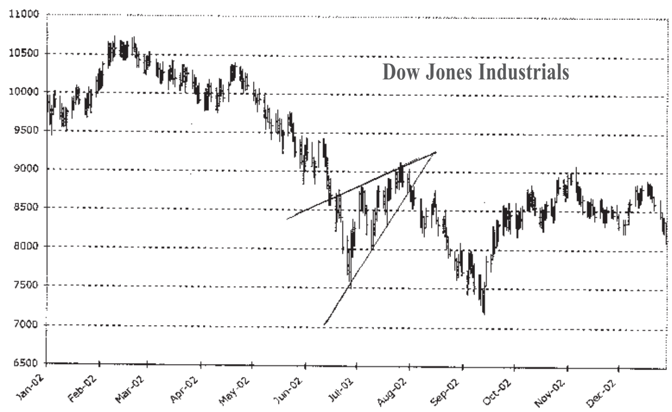


FIGURE 4: RISING WEDGE. The price chart of the Dow Jones Index shows a rising price pattern that might have a superficially bullish appearance. Unlike a rising channel, however, the upper line is not parallel to the lower one, indicating that the strength in buying is lacking. In other words, the selling arrives earlier and at a lower price compared to a healthy uptrend channel.

Rising (or Falling) Wedge: This is a rather common reversal pattern that differs in a subtle way from the pennant and symmetric triangle formations which we discuss as part of the two group patterns below. The price rises into the wedge which has positive slope (i.e., the rising wedge in Figure 4 and analogously for a falling wedge in Figure 5) as opposed to the symmetrical triangle or pennant which have zero or negative slope respectively. The negative prediction rendered by the rising wedge is evident upon a comparison with a rising channel. The rising wedge is essentially a deteriorating channel as the progressive new highs are unable to maintain the same ratios as the new lows (on the bottom trendline), thereby indicating that selling occurs earlier than one would expect, and sending a signal that a topping out process may be underway.

Two-Group Patterns

Symmetrical Triangles and Pennants: A common pattern in charts appears as a series of oscillations with diminishing amplitude. A pennant is formed by the two lines which join the successive tops and bottoms respectively. If the two slopes have equal magnitude and different sign, then the pattern is called a symmetric triangle (Figure 6). In general, pennants are regarded as consolidation formations which appear at regular intervals as a trend takes a pause. However, they can also be a reversal formation (Figures 7, 8) as shown in the next section. A breakout usually occurs before the vertex is reached as the trend is resumed. The pennant sometimes points down (i.e., the bisector of the two enveloping lines has negative slope) during an uptrend and points up in a downtrend. Volume is often diminishing until shortly before the breakout point, at which time it increases dramatically.

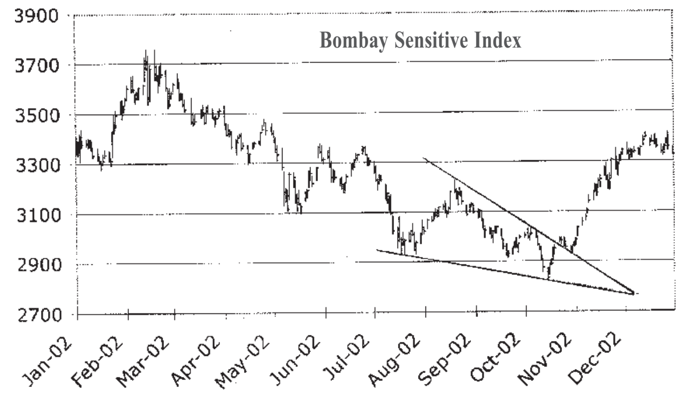


FIGURE 5: FALLING WEDGE. The falling wedge is analogous to a rising wedge in that generally declining prices camouflage increased buying at a higher price than would be indicated by a downward channel. The differential equations approach will show that this pattern is attained from oscillations around a rounded top.

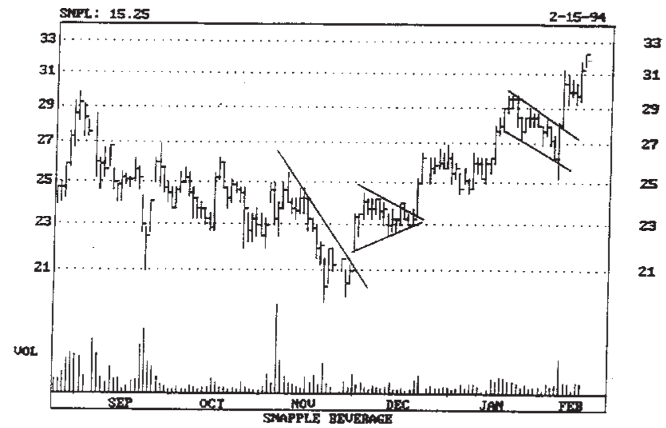


FIGURE 6: SYMMETRIC TRIANGLE AND FLAGS. Rapidly rising stock prices often exhibit *symmetric triangles* that occur at somewhat regular intervals in an uptrend or downtrend. In Snapple's stock, the two *symmetric triangles* occur after sharp rises in early December and early January, indicating that some selling has come in as a result of some investors deciding that it is time to take profits, due to valuation or excess upward movement. Once these investors exhaust their stock, however, the rally continues. A *flag pattern* develops in late January as a similar *consolidation pattern.*

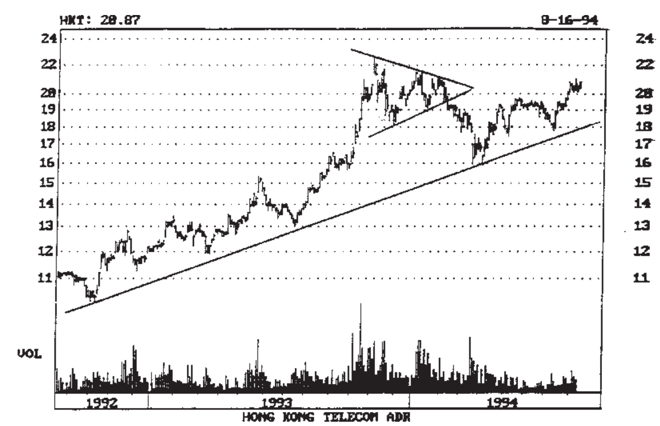


FIGURE 7: SYMMETRIC TRIANGLE AS REVERSAL (HONG KONG TELECOM). A strong speculative rally often terminates with a *symmetric triangle* that appears to be very similar to the *consolidation pattern* except that prices break out in the opposite direction. Hong Kong Telecom (HKT) received speculative attention not only because of its role as a large company in Southeast Asia, but due to the increasing activity in international telephone companies and the growing realization that HKT would tap the potentially huge Chinese market. All of these reasons brought a vast speculative pool of funds that eventually confronted the ample supply of more fundamentally oriented investors in the $19 to $21 range. The *symmetric triangle* took about 3 months to complete before a downtrend developed and terminated at a *double bottom.* Note that this stock also exhibits a *breakout from an upward trendline* in the second half of 1993, resulting in a small bubble that eventually is resolved through a *symmetric triangle,* so that both the evolution and the termination of the bubble are indicative of two distict groups of investors.

Flags: The flag is a consolidation pattern which is similar to the pennant in which the oscillation occurs between two parallel lines until a breakout occurs (Figure 6).

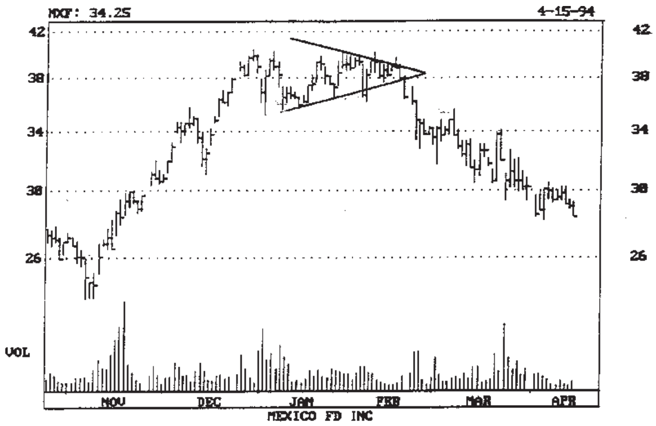


FIGURE 8: SYMMETRIC TRIANGLE AS REVERSAL (THE MEXICO FUND). A strong rally that took the Mexico Fund (MXF) from about $22 to $40 ran into resistance in the form of a *symmetric triangle* that encompassed six weeks. The rally began in mid-November 1993 as the House of Representatives' passage of Nafta assured its passage.

Breakout from Trendlines: The price moves initially within two parallel trendlines until one of the trendlines is broken. This provides an indication that the trend will continue in that direction (Figure 9). The significance of the breakout is related to the importance of the trendline that is broken, measured by the time duration of the trend and the number of times the price has touched and reversed at the trendline, and the decisiveness of the break, namely the percentage drop below the trendline.

3. The Price Equation and Basic Model

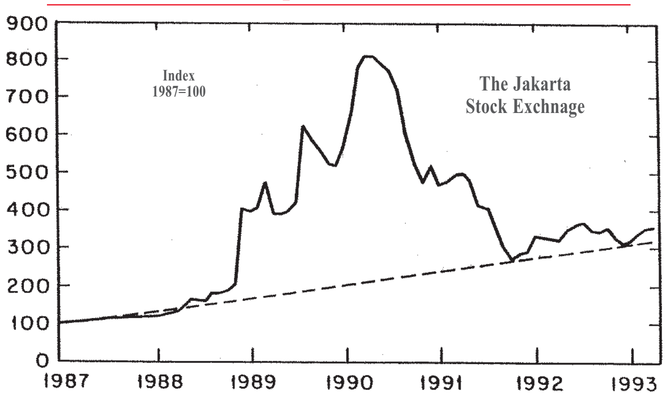


FIGURE 9: BREAKOUT FROM A TRENDLINE (THE FORMATION OF THE JAKARTA BUBBLE) Prices for the Jakarta Stock Exchange follow a steadily rising course until the end of 1989 when they spike upward and reach a peak in 1990 of eight times their 1987 value. After a collapse of the bubble, the prices find support at the original trendline.

The Theory of Price Adjustment

The classical theory of adjustment [see e.g. Watson and Getz (1981)] stipulates that relative price change occurs in order to restore a balance between supply, *S*, and demand, *D*, which in turn depend on price, i.e.

$$\frac{d}{dt}\log P = \frac{1}{p}\frac{dP}{dt} = F\left[\frac{D(P)-S(P)}{S(P)}\right] = F\left[\frac{D(P)}{S(P)}\right] \tag{3.1}$$

where D(P)/S(P) – 1 may be defined as the excess demand (normalized by supply). The function F has the properties

$$F(1) = 0 \quad \text{And} \quad F' > 0 \tag{3.2}$$

The vast majority of the phenomena discussed in the previous section cannot be explained on the basis of classical economics. We attempt to generalize the classical theories by preserving as much of the foundation as possible, e.g. the structure of the price equation, while modifying some of the concepts that are in clear conflict with the experiments and phenomena. The laboratory experiments such as Porter and Smith (1989) exhibit a very strong autocorrelation

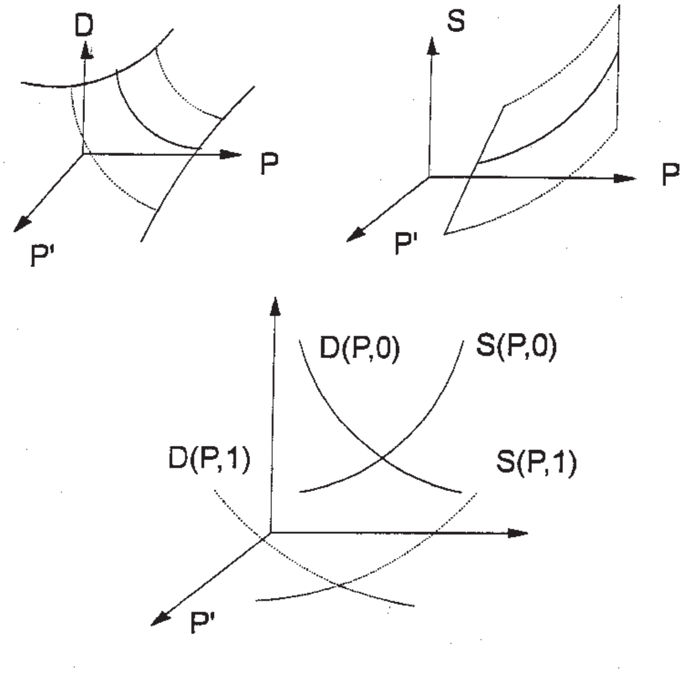


FIGURE 10: The classical supply and demand curves are modified so that each depends on the price derivative, P', as well as the price. Thus we have a two-dimensional surface for demand $D(P, P')$ and similarly for supply $S(P, P')$. At a particular price, P, demand increases and supply decreases as P' increases. The point $(P_a, 0)$ is an equilibrium if P_a is the fundamental value, and consequently satisfies $D(P_a, O) = S(P_a, 0)$.

[see also Caginalp and Balenovich (1994b) Appendix B] and are difficult to explain with supply and demand that depend only on price. Furthermore, there is a close mathematical connection between price derivative dependence and the presence of oscillations. The theory of ordinary differential equations [Coddington and Levinson (1955)] implies the absence of any oscillations for equation (3.1) for any reasonable (from an economic perspective) functions D(P) and S(P). The use of non-monotonic or non-convex supply and demand curves will not alter this conclusion. Hence, (3.1) is modified into the form

$$\frac{d}{dt}\log P = F\left(\frac{D(P,P')}{S(P,P')}\right) \tag{3.3}$$

Thus, the two-dimensional excess demand versus price curve is replaced by a three-dimensional graph (Figure 10). Monotonic supply and demand curves are adequate for our purposes so that there is a unique equilibrium point on the P′ = 0 plane. Note that for $P' \neq 0$ the points (P, P′) such that D(P, P′) = S(P, P′) cannot be on the evolutionary path since it would imply P′ = 0 which contradicts the assumption. The classical concepts of elasticity of supply and demand are easily extended to elasticity as a function of the price derivative. The elasticity in terms of the derivatives, written as d(log Q)/d(log P), where Q is quantity, represents the slope in the (log *P*) direction and expresses the extent to which rapid declines in price inhibit demand and encourage supply of stock submitted to the market. This picture is modified somewhat for our purposes of considering a finite asset base rather than a steady flow of supply and demand that might be more appropriate for a very long term perspective of the aggregate market. In either case, however, the convergence to an equilibrium price is significantly more complicated than the classical one in which the price converges exponentially to the unique equilibrium value without any oscillations.

A Supply/Demand Model

Equations (3.1) and (3.3) both express the idea that the rate of price change depends on the demand/supply ratio of stock. This is in fact reasonable not only from a microeconomic perspective but from the actual nature of market making in the exchanges in which many of the orders are placed at "market" and receive priority over "limit" orders. Thus, in an effort to maintain orderly markets, the specialist may buy or sell for his own account with a price determined by the demand/supply. We consider the total assets involved in trading and normalize it to unity. Each unit of assets is invested in stock or cash so that B is the fraction in stock while (1-*B*) is in cash. This is a simplification of a system in which transitional states are also considered [see Caginalp and Balenovich (1991)].

The change in price is determined by the supply and demand for stock. The total demand for a stock is given by the amount of funds in cash multiplied by the probability, *k*, that someone with a unit of cash will place an order to purchase stock (or equivalently the rate *k* at which cash flows into stock) and similarly for the total supply of stock, so that one has

$$D = k(1-B) \quad \text{and} \quad S = (1-k)B \tag{3.4}$$

We normalize *k* so that it assumes values between 0 and 1.

These simple equations, which are crucial to a complete dynamic theory, serve to translate the supply/demand issues into a ratio of buying and selling. An analysis of asset flow on the conservation of total capital implies

$$\frac{dB}{dt} = k(1-B) - (1-k)B + B(1-B)\frac{1}{P}\frac{dP}{dt} \tag{3.5}$$

This equation states that the fraction of assets in stocks changes in accordance with stock purchases, stock sales and stock appreciation respectively.

Note that equation (3.3) is often used in the linearized form $P' = (D/S-1)$. The linearized form is inadequate for global results because it (a) allows negative prices (b) requires a reference price to make it dimensionally correct, and (c) is not symmetric with respect to excess supply and excess demand. The use of relative price, i.e. P' replaced by $P^{-1}P'$, remedies the first two problems. The linearity of excess demand/supply, namely (c), means if $D = 0$, $S \neq 0$ then $P' = -1$ while $D \neq 0$, $S = 0$ implies $P' = \infty$. Hence linearity implies an extreme asymmetry with respect to demand and supply when D/S deviates significantly from unity. The logical requirement, $F(D/S) = -F(S/D)$, is satisfied by the logarithm, i.e. $\log(D/S) = -\log(S/D)$.

In fact, the most general function satisfying $F(D/S) = -F(S/D)$ can easily be shown to be any odd function of the logarithm. The difference between the logarithmic and the linearized equations is not significant when the excess demand or supply is not extremely large and simply amounts to rescaling the parameters.

Using the expressions (3.4) for supply and demand in the price equation (3.3) one then obtains

$$\frac{d}{dt}\log P = \log\left[\frac{k(1-B)}{(1-k)B}\right] \tag{3.6}$$

At this stage, if one used the classical assumption that buying/selling occurs only as a consequence of under/over-valuation (so that k depends on *P* but not P' then one would recover a modification (involving finite assets) of the classical theory which implies a return to the equilibrium value without any oscillations. Mathematically, this is a consequence of the first order nature of the equations. Note that equation (3.5) implies a finite size effect which is nontrivial even in the absence of trend based investment.

We use the notation $P_a(t)$ for the equilibrium point on the $P' = 0$ plane of the supply-demand curve so that $P_a(t)$ is the intrinsic value. In other words, the price at which supply and demand curves would intersect on a classical graph with no derivative considerations is represented by $P_a(t)$. In the experiments we discuss, for example, $P_a(t)$ unambiguously the computed value of the financial instrument based on the total payout.

A Price Derivative Dependent Preference Function

To complete the system of equations we consider a microeconomic derivation for *k* as a function of *P* and P'. If *k* depended only on the fundamental value $P_a(t)$, then one would have a generalization of the classical theory only in terms of the finiteness of assets and delay in taking action. We let ζ be defined as investor sentiment, or preference for stock over cash. To motivate the dependence of ζ on the history of price change, we consider the motivation of an investor who owns the security as it is undervalued but still declining. The choice available to this investor is either to sell or to wait in the expectation that those with cash will see the opportunity to maximize their profits by purchasing the undervalued security. The issue of distinguishing between self-maximizing behavior and reliance on optimizing behavior of others is considered in the experiments of Beard and Beil (1994) on the Rosenthal conjecture (1981) that showed the unwillingness of agents to rely on others' optimizing behavior. In these experiments, player A can choose a smaller payout that does not depend on player B, or the possibility of a higher payout that is contingent on player B making a choice that optimizes B's return (otherwise A gets no payoff). The experiments showed that A will accept the certain but smaller outcome that is independent of B. However, reinforcing a line of reasoning that is compatible with Myerson's (1978) "proper equilibrium" in which "mistakes" are related to the payoff consequences, Beard and Beil (1994) showed that player A will be less reluctant to depend on B when "deviations from maximality become more costly for B" (p. 257).

Applying these microeconomic principles to an undervalued [$P(t) < P_a(t)$] but still declining security [$dP/dt < 0$] in which players A own the security while players B have cash, the preference of A will depend upon the magnitude and duration of decline as well as the extent of the undervaluation. In particular, the longer and steeper the decline, the more evidence A receives about the unreliability of B. However, as the undervaluation increases, the Myerson aspect increases A's confidence in B and consequently A's preference for stock.

We take the total investor sentiment or preference function, ζ, as the sum of ζ_1 and ζ_2 where the former involves the trend and the latter valuation. In each case the basic motivation is summed with a weighting factor that declines as elapsed time increases.

We define the trend based component of investor preference as a continuous limit of a weighted sum of price changes, i.e.

$$\zeta_1(t) = q_1 \int_{-\infty}^{t} c_1 e^{-c_1(t-\tau)} \frac{1}{P(\tau)}\frac{dP(\tau)}{d\tau} d\tau \tag{3.7}$$

where q_1 is the amplitude and c_1^{-1} is a measure of the time scale or "memory length" for the trend. Hence ζ_1, is essentially an exponential moving average of the trend. So ζ_1 will be positive if the relative change $P^{-1}dP/dt$ has been mainly positive "recently." We assume similarly that the value based component, ζ_2 is a sum over the relative discount, i.e., $[P_a(t) - P(t)] / P_a(t)$, weighted with an exponentially declining function of time so that

$$\zeta_2(t) = q_2 \int_{-\infty}^{t} c_2 e^{-c_2(t-\tau)} \left[\frac{P_a(\tau) - P(\tau)}{P_a(\tau)}\right] d\tau \tag{3.8}$$

This means that the longer the discount persists, the greater will be the likelihood of investors acting upon it.

Since *k* denotes the probability that an investor with cash will invest it in stock, equivalently, the rate at which cash flows into stock, it is reasonable to expect that or, the rate (or probability) of selling a unit of stock is 1- *k*. Then *k* will be a function of ζ such that *k* is 1/2 (i.e. neutral probability) when ζ is 0. While $k = \zeta + 1/2$ would be adequate for practical purposes, it is desirable to use a function that maps the range of ζ [namely, $(-\infty,\infty)$ into [0, 1] to maintain the logical interpretation as a probability or rate. The simplest function that will accomplish this is

$$k(\zeta) = \frac{1}{2}\left[1 + \tanh \zeta\right] \tag{3.9}$$

Using any other smooth function would make very little difference as it would essentially rescale the basic parameters.

We emphasize that the nonlinearities in the log and tanh functions are not particularly significant since one has $\log(1 + x) \cong x$ and $\tanh x \cong x$ for small *x* and both functions will have arguments near these values for the cases of interest. In fact, any reasonable microeconomic derivation of k that has a trend based component would be linear in that component for sufficiently small values. Without a trend based component, a derivation of *k* would be incompatible with the laboratory experiments such as those of Porter and Smith (1989).

However, the nonlinearities expressed in (3.4) - (3.6) such as the product kB are essential since they provide the crucial idea that the total demand is the cash supply times the rate (or probability) of buying.

Differentiating ζ_1 and ζ_2 in expressions (3.7) and (3.8) results in the expressions

$$\frac{d\zeta_1}{dt} = c_1\left[q_1 \frac{1}{P}\frac{dP}{dt} - \zeta_1\right] \quad \text{and} \quad \frac{d\zeta_2}{dt} = c_2\left[q_2 \frac{P_a(t) - P(t)}{P_a(t)} - \zeta_2\right] \qquad (3.10, 3.11)$$

Hence, equations (3.4) - (3.6), (3.9) - (3.11) specify the complete system of ordinary differential equations which can easily be studied numerically on a personal computer with readily available software.

The parameters q_1, q_2, c_1, and c_2 are the only parameters in the system in addition to the scaling of time. Increasing q_1 tends to increase the importance of trend based investing and the amplitude of oscillations. Increasing q_2 tends to drive prices closer to the fundamental value. While the value of c_2 does not usually have a very dramatic effect on the price evolution, the numerical studies indicate that large values of c_1 (i.e. focusing on short term trends) can lead to unstable oscillations.

A Simpler Model in the Long Time Scale Limit

The four parameters can be reduced to only two by considering the long time scale limits. Letting c_1 and c_2 approach zero while $F_1 = c_1q_1$ and $F_2 = c_2q_2$ are held fixed, one obtains in place of (3.10) and (3.11) the equations

$$\frac{d\zeta_1}{dt} = F_1 \frac{1}{P}\frac{dP}{dt} \quad \text{and} \quad \frac{d\zeta_2}{dt} = F_2 \frac{P_a(t) - P(t)}{P(t)} \qquad (3.12, 3.13)$$

The resulting system (3.4), (3.5), (3.6), (3.9), (3.12), (3.13) can then be studied and compared with experiment with only two parameters. This method was implemented in a previous paper [Caginalp and Balenovich (1994b)] in which the theoretical results were compared with the experiments of Porter and Smith (1989). The procedure consisted of using any one of the experiments to evaluate F_1 and F_2. The other experiments could then be predicted by the model with no adjustable parameters. The statistical comparisons showed that the model, with these values of F_1 and F_2, was decisively more accurate than the efficient market hypothesis (statistical significance of 1×10^{-12} or the model with F_1 set to zero, thereby confirming the necessity for the trend based term (statistical significance of 1×10^{-8}). Similar results are obtained for the four parameter system.

Discussion of Assumptions

Throughout this analysis we focus on the active investors who form a small fraction of the total financial system but determine the price through their demand/supply ratio for stock. Thus, the total amount of stock owned by the active trading group is not constant, although the total amount of stock in the system may be conserved. The total assets of the investors is assumed to be constant except due to changes in stock price, so that the investors are not experiencing a change in external resources. This assumption is made simply to focus on the essential issue, since a source term in (3.5) can easily account for an increase in wealth.

The model we discuss does not prejudice *a priori* in terms of the extent to which markets are efficient. The full range of possibilities is in fact covered by the range of parameter values of q_1, c_1, q_2, and c_2.

The numerical computations confirm that if the trend based coefficient is sufficiently small, then the price evolves rapidly toward $P_a(t)$ with little or no oscillations. This corresponds to a classical rational expectation model. As q_1 is increased, the damped oscillations increase in magnitude and frequency. Above a critical value of q_1 the oscillations become unstable in the sense that they increase in magnitude without bound. The behavior associated with the experimentally determined values of q_1 and q_2 falls in the damped oscillation regime. This pattern of behavior occurs for a broad range of functions $P_a(t)$, including functions with an abrupt drop. Consequently, the extent of trend based investing is decided by the data, as the model is sufficiently robust to describe the spectrum from purely value based investing to speculative markets.

Multiple Group Generalization

The principles used above can be generalized to a set of disparate investor groups labeled 1, 2, ..., m with assets normalized at unity. The assets of investor group j which are in stocks are denoted by B^j. Each group has its own preference variables ζ_1^j and ζ_2^j, analogous to (3.7) and (3.8), and its own transition ratios, $k^j(t)$, given by

$$k^j = \frac{1}{2}\left[1 - \tanh\left(\zeta_1^j + \zeta_2^j\right)\right]. \qquad (3.14)$$

Each group may also have its own assessment of value $P_a = P_a^j(t)$ yielding

$$\frac{d\zeta_1^j}{dt} = c_1^j\left[q_1^j \frac{1}{P}\frac{dP}{dt} - \zeta_1^j\right] \quad \text{and} \quad \frac{d\zeta_2^j}{dt} = c_2^j\left[q_2^j \frac{P_a(t) - P(t)}{P_a(t)} - \zeta_2^j\right]. \qquad (3.15)$$

The fraction of assets of group j which are invested in the stock is described by the analog of (3.5), namely,

$$\frac{dB^j}{dt} = k^j(1 - B^j) - (1 - k^j)B^j + B^j(1 - B^j)\frac{1}{P}\frac{dP}{dt} \qquad (3.16)$$

The price equation is determined as in the single group case, since the change in price depends only on the ratio of buy/sell orders, irrespective of the origin of these orders, so that one has

$$\frac{d}{dt}(\log P) = \log\left[\frac{\sum k^j(1 - B^j)}{\sum (1 - k^j)B^j}\right] \qquad (3.17)$$

Hence, the $(3m + 1)$ ordinary differential equations, (3.15) - (3.17), together with the *m* algebraic equations (3.14) provide a mathematically complete system which can be studied numerically upon specification of the initial conditions. The two-parameter version is generalized in a similar manner.

These equations can easily incorporate (i) the effects of additional cash that may be acquired by one or more groups, or (ii) the effect of additional supply of shares. An influx of cash into group *j* changes (3.16) by adding a term $-dN(t)/dt$ that represents the flow of new cash per unit time. Similarly the effect of adding new stock to the system can be considered as a term that is added to the supply portion of (3.17).

4. A Theoretical Foundation for Technical Analysis

The classical theories offer no explanation for the set of methods that are generally described as technical analysis. Some of the patterns such as trendlines may be partially explained as random fluctuations about a line of constant slope which represents the changing fortunes of the company or stock. However, this explanation clearly fails for pennants, wedges and other patterns. While the existence of such patterns must be verified by statistical methods, the modeling of these patterns is only possible if there is a mechanism for inducing oscillations in prices, as discussed in the derivation of the equations in Section 3.

Our central thesis is that all of the technical charts discussed in Section 2 can be obtained as a consequence of the simple models defined in Section 3. Thus, if one accepts the basic economic assumptions of the models of Section 3, which involve capital flow and both value and trend based investment, then one must accept the mathematical conclusion that all of these patterns arise from these economic assumptions.

Single-Group Patterns

We first describe the evolution of patterns which are categorized as a single group in Section 3. In each of these cases we use equations (3.4) - (3.6), (3.9) - (3.11) subject to an intrinsic price $P_a(t)$.

Trendlines or Channels: The conventional explanation for these patterns is that the potential buyers and sellers both move their bid and ask prices upward (in an uptrend) in a way that is approximately constant in time. This explanation is not completely satisfactory in that the seemingly regular oscillations are attributed to the random buying and selling by a third group of investors who have a smaller amount of capital than the major buyer or seller.

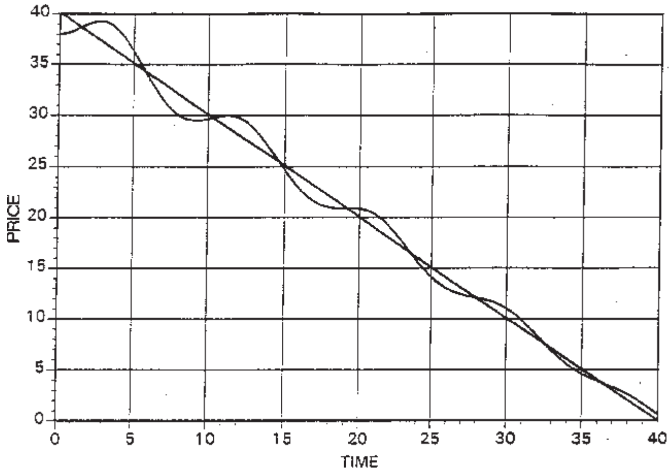


FIGURE 11: DOWNWARD TRENDLINE: The price oscillates about a linearly increasing $P_a(t)$ that represents steadily improving fundamentals. The magnitude of the oscillations depends on the trend based coefficient.

This theory offers a very simple and natural perspective which explains the regular oscillations between the lower and upper part of the trend channel. A downward trend is assumed to be a consequence of a steadily decreasing assessment of the value of the financial instrument (Figure 11). Thus, the assumption of a linearly decreasing $P_a(t)$ is compatible with classical economic theory. The intrinsic value function is given by $P_a(t) = -t + 40$. Using $P_a(t)$ in equations (3.10) - (3.11), we compute solutions with a set of initial conditions, including for example $P(0) = 38$, and the values of $q_1 = 950$, $q_2 = 100$, $c_2 = 0.001$. Other values for $P(0)$, q_1, c_1, q_2, c_2 yield similar results. Under these conditions, the price oscillates about $P_a(t)$, so that P(t) varies between two lines parallel to $P_a(t)$. The width of this channel is dependent on the values of the parameters. In general, a higher trend based coefficient favors larger oscillations and a larger channel, while a higher value based coefficient favors a stronger link to the intrinsic value and therefore smaller oscillations.

Within this model a price path similar to a trendline also forms when the investing is largely trend based and value investing plays a minor role ($q_2 << q_1$).

Rounding Top (or Bottom): Once again we make the assumption that is close to the central ideas of classical economics by assuming a $P_a(t)$ function which represents a simple turn-in-fortunes of the financial instrument, namely a linearly increasing $P_a(t)$ for $t < t_0$ and a linearly decreasing $P_a(t)$ for $t > 0$. The function in Figure 12 is given by $P_a(t) = t + 5$ for $t < 20$ and $P_a(t) = -t + 45$ for $t > 20$, with $P(0) = 4$, $q_1 = 820$, $c_1 = 0.001$, $q_2 = 44$, $c_2 = 0.001$ For values of q_2 which are smaller those found in trendlines we obtain a rounded top. This is essentially what is to be expected since the rounded top is not observed frequently in charts and is associated with markets in which volatility is low and value based investing is the dominant strategy. Examples of such markets include high-quality utility stocks and bonds in which the value is easily measured in terms of the dividend paid versus the prevailing interest rates and the financial health of the company, which is carefully evaluated and rated by Moody's or Standard & Poor's, etc. Hence, under these conditions, we do not expect large oscillations near the tops or bottoms, unlike the head and shoulders or double top formations, to be discussed below, which are characterized by a trend based strategy resulting in more complex reversal patterns.

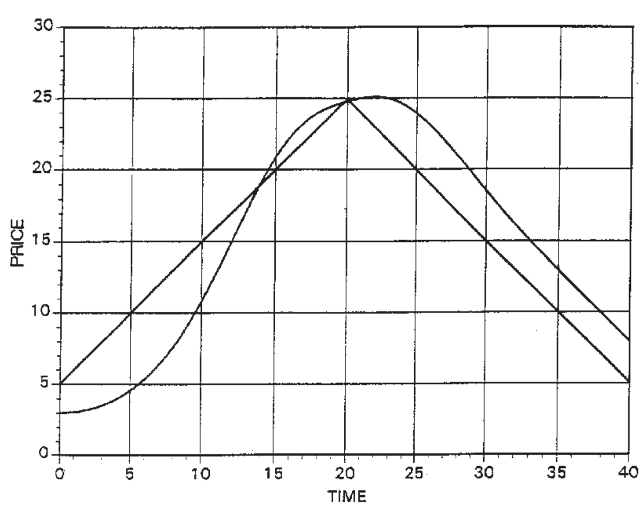


FIGURE 12: ROUNDING TOP: Prices follow an inverted V-shaped $P_a(t)$ closely due to a relatively small trend based coefficient. The vertex is smoothed due to the trend effect.

Key Reversals, V-formations and Spikes: Clearly, a V-formation will be obtained in a similar way to the rounding top if the value of q_1 is lowered and the value of q_2 is raised. Thus, using the same $P_a(t)$, P(0), c_1, and c_2 as in the rounding top discussion we let $q_1 = 700$ and $q_2 = 1000$ (see Figure 13). The result is that prices track $P_a(t)$ very faithfully. This topping formation is closest to the principles of classical microeconomics. A key reversal pattern appears on a typical stock chart as a long vertical line (representing the day's range), which is higher than those to the left and right of it, with a slash near the bottom (representing the closing price) and this vertical line is higher than those to the left and right of it. Of course, if we were to plot prices as a function of time (by minutes or hours) then the day of the key reversal typically contains a V-formation or a rounding top on a finer time scale. Hence, the key reversal is explained on the same basis as these simple formations on this time scale.

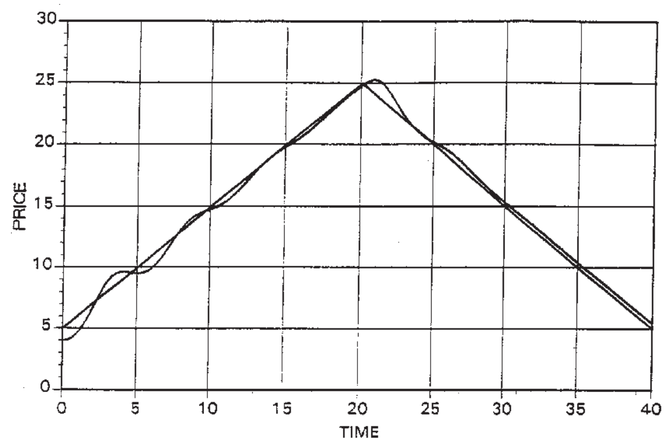


FIGURE 13: V-FORMATION TOP: An inverted V-shaped $P_a(t)$ is used as in the *rounding top* but now with a larger value based and smaller trend based coefficient.

Head and Shoulders: We make the same assumption on $P_a(t)$ as in the rounding top, namely $P_a(t)$ is first linearly increasing, then linearly decreasing. We use the same function $P_a(t)$ as in the rounding top with the values $P(0) = 5$, $q_1 = 1000$, $c_1 = .001$, $q_2 = 425$, $c_2 = .001$. By using these larger values for q_1 and q_2 we obtain a typical head and shoulders reversal or topping out pattern (Figure 14). As in the financial markets, one can obtain a variety of head and shoulders patterns, e.g., with the right shoulder slightly lower than the left shoulder. This variation is induced by the timing of the oscillations with respect to the peak time of $P_a(t)$. In particular, the extent of undervaluation or overvaluation $P(t_i)$, in the initial time, along with the magnitude of $t_0 - t_i$, where t_0 is the peak time for $P_a(t)$ will determine the precise shape of the head and shoulders top.

Similarly, a head and shoulders bottom, such as the one shown for the Dow Industrials in 1990 (Figure 3) is produced with the following parameters: $P_a(t) = -x + 3$ for $0 < x < 15$ and $P_a(t) = x$ for $15 < x < 30$ and $c_1 = .005$, $c_2 = .05$, $q_1 = 210$, $q_2 = 50$ as shown in Figure 15.

An important aspect of our conclusions is that a rather complex reversal pattern can be obtained as a consequence of a single group of investors with identical motivations and assessment of the value of the financial instrument. This may be in the sharp contrast to the initial appearance of a tug-of-war between disparate investor groups, or vacillating nature of the basic value of the financial instrument. Hence the conventional wisdom that market tops are generally complex formations, rather than a simple U-turn, is explained in terms of

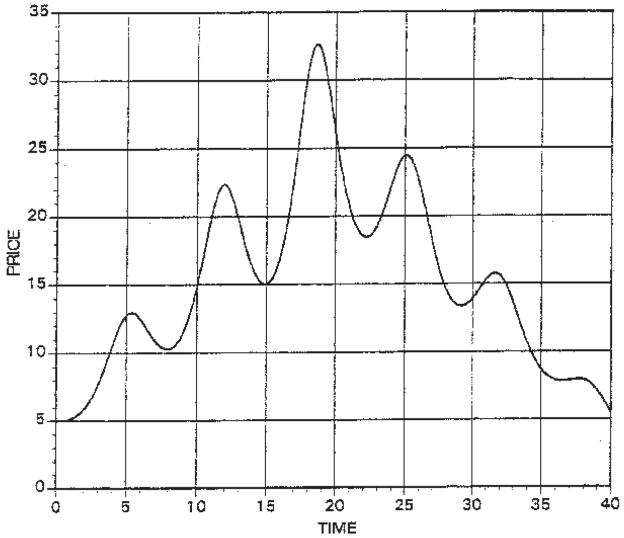


FIGURE 14: HEAD-AND SHOULDERS TOP: Using an inverted V-shaped function for $P_a(t)$, the peak is reached through a series of oscillations, with the highest peak near (and usually just after the peak in $P_a(t)$. This forms a classical *head and shoulders top.*

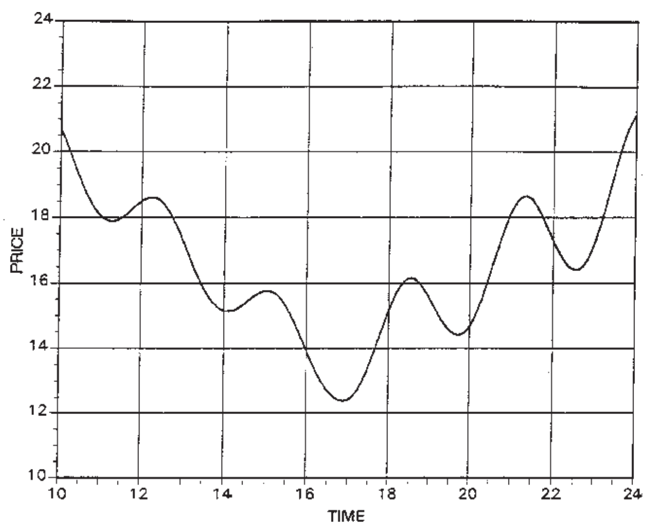


FIGURE 15: HEAD AND SHOULDERS BOTTOM: With a V-shaped $P_a(t)$ function, the price oscillates about the valuation with an absolute minimum that is close to that of $P_a(t)$. The precise timing depends upon the initial conditions. The two oscillations that accompany the bottom are the secondary support in the *(inverted) head and shoulders bottom.*

the coupling between natural market oscillations and the changing value of the company. In fact, a long standing idea of Wall Street is that when the trend becomes interrupted and begins to move into a region of large oscillations and slower average increases, then "distribution" has begun. This means that the more knowledgeable investors with large amounts of stock, and presumably among the early participants in the uptrend, have begun selling their shares into the rally. While this may be the origin of some topping formations, as we will see in the wedge formations below, it is clearly not a necessary condition in the formation of head and shoulders as well as the related multiple tops discussed later.

As an example which illustrates these ideas is the aggregate U.S. market in the second half of 1990. Soon after the Iraqi invasion of Kuwait in early August of 1990, the chances of the U.S. becoming involved in a war and the consequent recessionary effects on the U.S. economy implied a steady downward reevaluation of the intrinsic value of the major stocks. By October, the Federal Reserve's concern of a deepening recession led it to a more accommodative stance, so that the gradual decline in interest rates effectively provided a steadily increasing value for stocks (by reducing the value of competing fixed income instruments, and also by reducing corporate financing costs). At the same time the analysis of the possible war scenarios led to a general consensus that the outcome of a war would not be as grave as first thought for the U.S., since oil supplies would probably not be interrupted. Thus, the intrinsic value of the stock market, i.e., $P_a(t)$ in our terminology, can be represented by a V-shape. With this $P_a(t)$ in the equations, one obtains a $P_a(t)$ which is an inverted head and shoulders pattern similar the general pattern in late 1990 (see Figure 3). We note that the introduction of a more complex peak for $P_a(t)$ would necessarily introduce the more complex topping out patterns (than the head and shoulders) which are often observed in financial markets.

Double and Triple Tops (or Bottoms): The basis of this pattern is similar to the head and shoulders. We assume a $P_a(t)$ which is linearly increasing, then level and finally linearly decreasing. The intrinsic function is $P_a(t) = x + 5$ for $0 < t < 15$, $P_a(t) = 20$ for $15 < t < 20$, and $P_a(t) = -x + 40$ for $20 < t < 35$. Using the same values of P(0), c_1, and c_2 as those in the V-formations with parameters $q_1 = 950$ and $q_2 = 400$ we obtain a P(t) that is a double top formation. The features of $P_a(t)$ and the extent of initial undervaluation or overvaluation in P(t) determine the nature and number of oscillations near the peak. In particular, we have assumed that $P_a(t)$ is symmetric about the peak, then there are the following parameters: the magnitude of the slope in the first and last parts of $P_a(t)$ is one, the length of the level part is 5, the magnitude of $t_0 - t_i$ is 17.5 (where t_0 is the midpoint of $P_a(t)$ and t_i is the initial point), and $P_a(t_o) - P_a(t_i)$ is 15.

Our analysis shows that the different shapes such as the double top and the head and shoulders can be the consequence of identical $P_a(t)$ with slightly differing undervaluation (in timing or extent). In fact, one can obtain a one-parameter family of topping patterns, including these two, by simply varying the timing of the undervaluation and thereby the timing of the oscillations with respect to the peak of the V-shaped $P_a(t)$. It is also clear why the triple top is rare in markets, since a V-shaped $P_a(t)$ does not result in such a top with a wide range of parameters and initial undervaluations. A triple top can be attained with a $P_a(t)$ function which has a longer flat top, and using parameters in which the trend based coefficient is rather small.

Two-Group Patterns: A number of patterns arise as a consequence of the interactions between two or more groups with differing assessments of value and/or different motivational characteristics. These types of patterns can be expected in markets that are very speculative and/or highly visible in that they attract new groups of investors. The origin of many of the bubbles in world markets can be explained (as elaborated below) with the ansatz that a new group of investors acquires an interest (or the money to invest) in a security that has been traded by more experienced investors. The new group has either little idea of the true value (so that q_2 is small) or has an inflated estimate of it (so that $P_a^2(t)$ is high). Just as the influx of their funds creates a bubble, the finiteness of their assets terminates it, and is accelerated by the trend based investing. We use the equations (3.14) - (3.17) in each of the circumstances discussed below.

Breakout from a Trendline or Channel: Recall that a channel is essentially a series of oscillations about a linear $P_a(t)$ which we assume for the sake of concreteness is increasing. If we assume that Group 1 has the characteristics that produce the channel (described earlier) then we define Group 2 as investors with a higher trend based coefficient and/or a higher assessment of value. Furthermore, we let Group 2 acquire cash during the trendline formation. The result (Figure 16) is that a break occurs in the upper trendline and the channel gives way to a bubble. If we compare this microscopic explanation with a typical bubble, e.g. Indonesia in the late 1980's (Figure 9), we see that a clear lower trendline is in effect during the entire process. This trendline is formed by the more sophisticated and value-oriented investors who are bidding in accordance with a value that is steadily increasing at a sustainable rate due to continued growth and increasing earnings. The steady increase in price, however, draws the attention of potential investors who may have little or no experience. These investors are not only mesmerized by daily price changes, but they tend to confuse projections of what the stock should be trading say three years from now with what one should pay for it today. For example, if the stock has been increasing at 20% per year, the price is currently at $10 and the investor plans to use the money in three years, the idea of paying $15 now

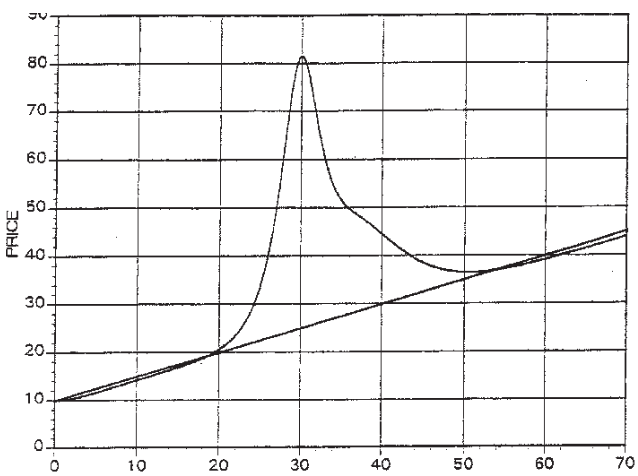


FIGURE 16: BREAKOUT FROM A TRENDLINE: Two groups of investors are involved in trading an equity that has steadily increasing fundamentals. Group 1 is largely value based and has an estimation of value, $P_a^1(t)$, that is linearly increasing with time. Group 2 has a constant estimate of value, $P_a^2(t)$, that is near the high range of the linear function of Group 1, and also has a much higher trend based coefficient. Initially, Group 1 has almost all of the assets. Near the middle portion Group 2 acquires assets that are about one-third of the total. Prices rise dramatically as this group begins purchasing stock and following the trend. Group 1 is not eager to sell to early, as they are also influenced by the trend. As Group 2 exhausts its cash supply, prices begin to plummet. The rout continues until Group 1 resumes value oriented purchases.

seems reasonable to those investors. However, they have a finite supply of cash and ultimately the cash is inadequate to buy the increasing amounts of stock that Group 1 is selling due to the overvaluation. As supply of stock overwhelms demand the price begins to fall and Group 2 now begins to sell for the same reason that one induced them to buy — the trend in stock price — and the bubble bursts. Note that Group 1 is also somewhat trend oriented, and in fact it would be irrational to sell now if it appears that some investors will pay more for it later.

Symmetric Triangles and Pennants: The basic origin of these formations is the presence of two or more groups with distinct assessments of value. The pattern is essentially a series of oscillations about a particular value that represents the assessment of one group on fundamental value. The issue of whether the pattern is a consolidation or a reversal is ultimately decided by the exhaustion of either cash or stock on the part of the net buyers or sellers, and is not obvious from the immediate appearance of the triangle, though a more careful examination of prices may provide a better indication.

An example of a symmetric triangle that is a consolidation can be created by distributing 80% of the initial assets to Group 1 which assesses true value at

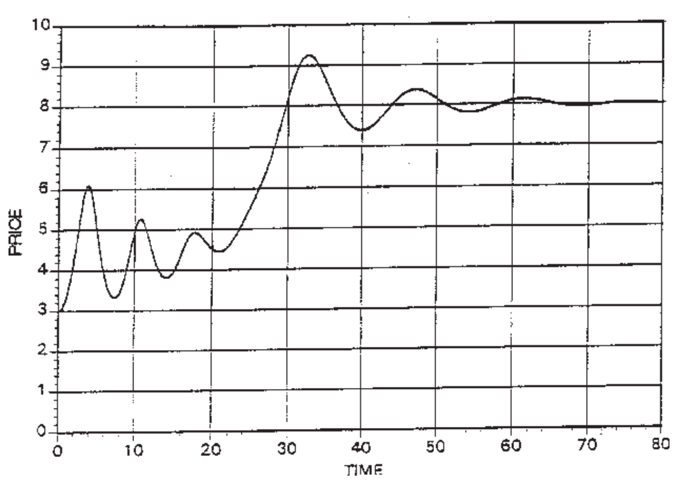


FIGURE 17: SYMMETRIC TRIANGLE: Two groups have asymmetric information or opinion on the value of the security, $P_a^1(t) = 8$ and $P_a^2(t) = 4$, that are constant in time. Initially, Group 1 has 80% of the assets, while Group 2 has the remainder. As prices move beyond $P(t) = 4$, Group 2 becomes a net seller, though they continue buying and selling in part with the trend. At some point, as the vertex of the triangle is approached, this group exhausts its supply of stock, thereby creating a shortage of supply that sends prices upward until Group 1 begins selling above their estimated value of 8.

$P_a^1 = 8$, while Group 2 values it at $P_a^2 = 4$. With the other parameters set at $c_1^{(1)} = 0.001$, $q_1^{(1)} = 950$, $c_2^{(1)} = 0.001$, and $q_2^{(1)} = 100$ for Group 1 and $c_1^{(2)} = 0.001$, $q_1^{(2)} = 900$, $c_2^{(2)} = 0.001$, and $q_2^{(2)} = 1000$ for Group 2, one obtains the formation shown in Figure 17. The essential feature of combination does not change if the parameters $q_1^{(i)}$, $q_2^{(i)}$, $c_1^{(i)}$, $c_2^{(i)}$, i = 1 or 2, are altered. The amplitude of the oscillations in the triangle depends chiefly on the trend based coefficient of the group whose assessment of value is within the triangle, and secondarily on the group with the higher assessment.

The continuation triangle can be changed to a reversal pattern (at the higher value) by exchanging the fraction of assets from one group to the other.

An Example in Detail: The Symmetric Triangle as a Reversal Pattern

An examination of the Mexico Fund from late 1993 to early 1994 shows a rapid rise from mid-November until January. A long, symmetric triangle is then formed and prices break out of this triangle in mid-February and begin a steady downtrend. The Mexico Fund is a closed-end fund that invests rather successfully in a diversified manner in Mexican equities, and is affected by news that impacts Mexico. These include the U.S. House passage of the Implementation Act in mid-November (where the outcome was in doubt to many), the peasant rebellion on January 1, 1994, and the assassination of the leading candidate for President on March 23, all of which were unprecedented in modern Mexican history. While the rise certainly appears to be precipitated by NAFTA, neither the rebellion nor the assassination appears as the immediate cause of the downtrend that begins in mid-February. In fact, this date marked a small scandal in Brazil that drove a number of Latin American markets down, perhaps with the explanation that investors realized that political uncertainty remains a possibility in Latin America. From a fundamental point of view, this explanation is no more convincing than the factor of (roughly) two variations in the price of this stock in a short time period. Other deviations from classical theory include the variations in the discount/premium (20% discount to 5% premium) and the drop in the stock during the September rights offering that added more shares.

Our explanation of this price pattern of this stock is based simply on the existence of two groups with differing assessments of value (i.e., asymmetric information). As far back as June 1994, published reports indicated that some large investors felt certain of passage and estimated that the Mexican Bolsa could rise from 1900 to 2400 with the passage. This would probably translate to a value of about 32 or 33 on the Mexico Fund. This establishes a fundamental value for this group. On the other hand, another group of investors, who probably had not thought of investing in Mexico until the intense debate on NAFTA, probably were much more conscious of the trend, and their vague assessment of value probably placed the fundamental value at a somewhat higher number, perhaps 38 or 39. An examination of volume shows that volume increased significantly in the months after the House passage and returned to previous levels afterwards (Figure 8).

It may also be reasonable to assume that the sophisticated group will have greater assets or at least will have inventory to sell after the enthusiasm of post-NAFTA buying has faded. Of course, it is difficult to know for certain the precise characteristics of various investor groups. However, our approach allows for the testing of a particular set of assumptions to determine whether it is consistent with a particular pattern.

Toward this end, we make a very simple set of assumptions and examine the extent to which they can be varied without altering the qualitative nature of the pattern. We let Group 1 have $P_a^1(t) = 32$ while Group 2 has $P_a^2(t) = 39$. The other parameters are $c_1^{(1)} = 1.00$, $c_2^{(1)} = 0.001$, $q_1^{(1)} = 0.54$, $q_2^{(1)} = 900$ for Group 1 and $c_1^{(2)} = 1.00$, $c_2^{(2)} = 0.40$, $q_1^{(2)} = 0.01$, $q_2^{(2)} = 800$ for Group 2. The result for the price evolution is displayed in Figure 18. An explanation of this evolution is that both groups initially buy due to both undervaluation and positive trend. As the price becomes very high, however, Group 2 begins some selling. After a few oscillations the price appears to be very stable and just below 60. Gradually, Group 2 sells as Group 1 buys, both due to valuation. Perhaps the most

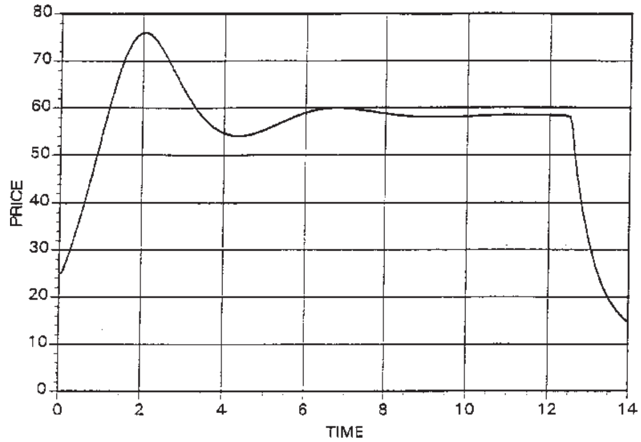


FIGURE 18: TRIANGLE REVERSAL INVOLVING TWO GROUPS: An initially undervalued security rises sharply as Group 1, with the lower estimate and larger assets (60% of total), sells as the rally matures, while Group 2, which is more trend oriented continues to buy. As Group 2 exhausts its cash supply, prices plummet precipitously since both groups have strong reason to sell.

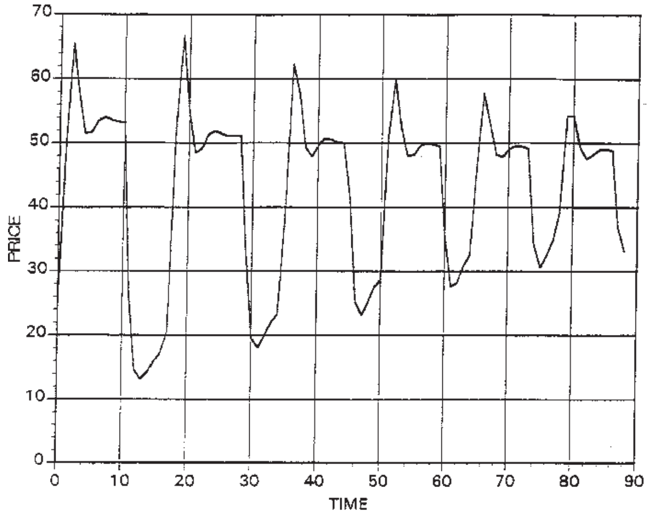


FIGURE 19: CONTINUATION OF TRIANGLE REVERSAL WITH TWO GROUPS: The pattern of the first bubble is repeated though the amplitude shrinks with each cycle and gradually converges to a steady state that is determined by the fundamental valuation of each group, the initial conditions and most significantly, the relative assets of the two groups.

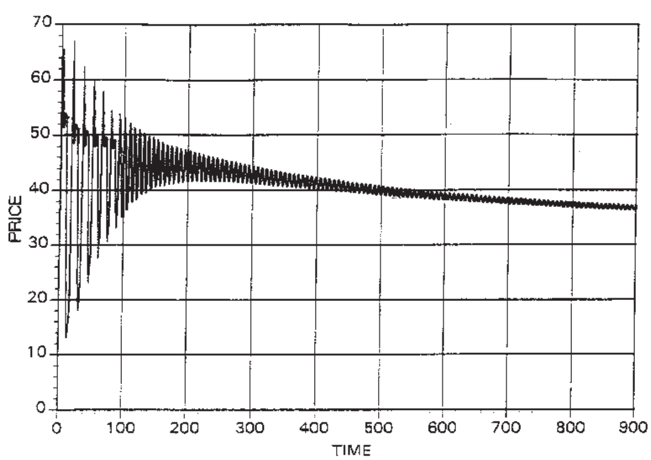


FIGURE 20: LONG TERM BEHAVIOR OF PRICE WITH TWO GROUPS: Finally, as the differential equations evolve for a very long time, we see that an economic equilibrium is achieved. The precise value depends on the parameters and initial conditions, most significantly the relative assets of the two groups with differing assessments of value. Roughly speaking, the group with larger assets tends to impose its viewpoint upon (while profiting from) the other group.

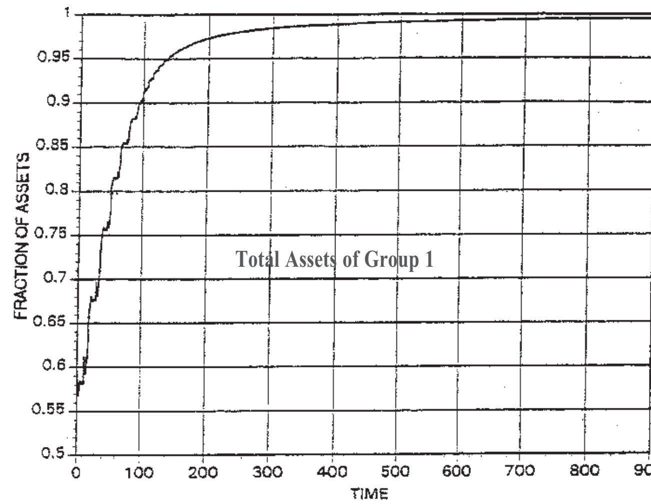
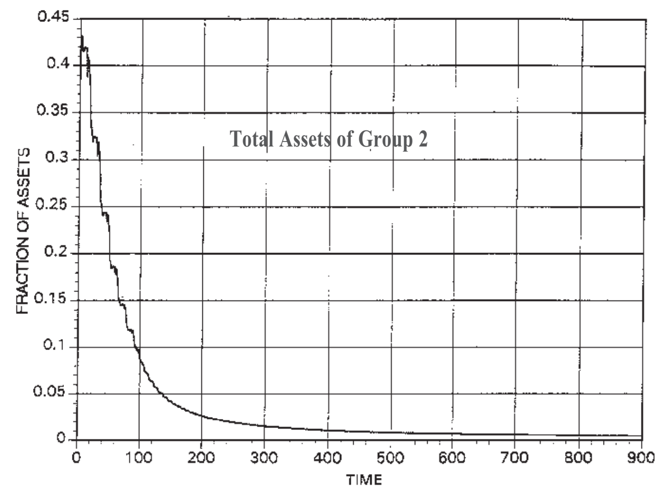


FIGURE 21 A, B: TOTAL ASSETS OF GROUP 1, GROUP2: The two groups experience very different fortunes during these oscillations. Group 1, which has 60% of the assets at the outset, gradually acquires 100%. Group 2 steadily loses all of their assets. The market in this case is an efficient way for one group to steadily acquire the assets of the other.



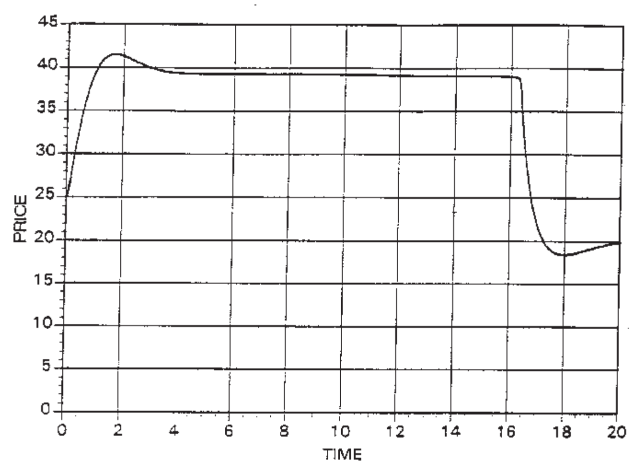


FIGURE 22: TRIANGLE REVERSAL WITH SMALL TREND BASED STRATEGY: Both Group 1 and Group 2 have small trend based parameters ($q_1^{(1)}$ and $q_1^{(2)}$). Initially the price rises very rapidly then approaches a quasi-equilibrium in which Group 1 is content to sell the stock to Group 2 at a price just below $40 where Group 2 believes it is a bargain. Hence, this false equilibrium is at essentially the highest price that Group 1 could obtain for it.

interesting point occurs when a sharp drop-off appears as almost a discontinuity in the derivative of price. At this point, Group 1 has exhausted their supply of cash to the point where their buying is inadequate to sustain the price. Once the decline begins, however, they participate in the selling due to the trend and the steep decline sets in.

If we allow the computations to continue we find that a similar cycle is repeated many times although one of the oscillations disappears (Figure 19). In time, the oscillations become much smaller and the price reaches a steady state that depends on the parameters and initial conditions (Figure 20). The attainment of this price is considerably more complicated than in the classical adjustment. An interesting feature of this process emerges in the plot of the fraction of the total assets owned by each group, as shown in Figure 21 A, B. Throughout this complicated process Group 2's assets increase almost steadily from 60% to almost 100% while Group 1 gradually loses its share.

Also, it appears that the most important factor in determining the steady state price is the fraction of assets of each group rather than the extent to which it emphasizes a value based investment strategy. The group with greater assets dominates the determination of the eventual price and so its strategy pays off more than the other. Furthermore, the numerics also indicate how a quasi-equilibrium can be formed at a price that appears to satisfy both groups — in the case of Figure 22 just below 40 — so that one group sells as the other buys, although this price cannot be maintained because one group exhausts its supply of cash. Sophisticated traders say that a period with low volatility often precedes a big move (up or down). From this perspective, it is clear that this phenomenon is caused by one group exhausting its supply of cash/stock before the other, in a manner that is reminiscent of a simple field battle of foot soldiers. The battle appears to be a stalemate as the front moves very slowly, until one of the armies is decimated and the front then advances rapidly. Consequently, if one can make the assumption that two distinct groups are involved in trading and both have a significant trend component, then a period with no oscillations indicates that the trading price is satisfactory to both groups from a value perspective. In fact, numerical studies with a lower trend coefficient for both groups display no oscillations near the two true value prices but a similar oscillation between the two values until a steady state is reached. This confirms a trader maxim that long periods with little change precede a big move up or down.

5. Japanese Candlestick Charts

An approach to technical analysis that has been used for several centuries in Japan, but has attracted interest only in the past quarter century in the West, is known as Japanese candlesticks [see, for example, Morris (1992)]. As in conventional technical analysis, some interpretation (or statistical criteria) becomes necessary to decide whether or not one is in a trend. Beyond this, however, the rules can be stated completely with several inequalities for each pattern. This method incorporates the additional information of the opening price, as well as the close and range, thereby giving a clearer indication of the short term trend. The graphs consist of a series of three types of symbols: the white body, black body and the cross or doji line as shown in Figure 23.

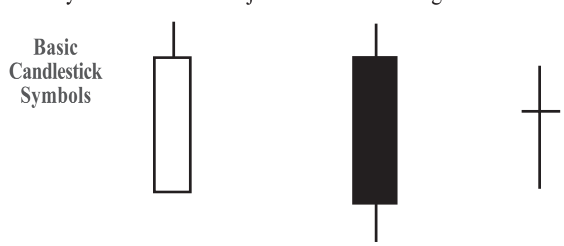


FIGURE 23: THE BASIC CANDLESTICK SYMBOLS: A candlestick chart consists of a series of three types of symbols. A *white (or open) body* signifies that the closing price (which is the value at the top of the rectangle) was higher than the opening price(which is the price at the bottom of the rectangle). The lines above and below the rectangle represent the total trading range during the day. A *black (or closed) body* signifies a day in which the opening price (at the top of the rectangle) exceeds the closing price (at the bottom of the rectangle). The *shadows* or lines extending beyond the rectangle again signify the trading range. A *doji or cross* signifies an opening which is equal to the closing price, represented by the horizontal line, while the vertical line represents the trading range.

The bottom of the white body is the opening price while the top is the close. The opposite is true of the black body, while the doji indicates an equal opening and closing price as shown by the horizontal line. In all three cases, the lines above and below the body, called the shadows, indicate the trading range during the day.

The main objective of candlestick analysis is to provide an indication of market reversals on a short time scale of about one to five days, and to distinguish these from continuation patterns. As with other forms of technical analysis the patterns are grouped into reversal and continuation patterns. For candlesticks, however, there is also some indication of the confidence level of the prediction as a result of grouping into three categories: (a) no confirmation is necessary, (b) confirmation is suggested, (c) confirmation is necessary. Confirmation is defined as an additional day which supports the conclusion of the pattern, namely continuation or reversal.

Most of the implications of the candlestick analysis are a natural consequence of the asset flow approach. The main difference between the conventional analysis discussed earlier and candlesticks is that the latter concerns a much smaller time scale. Consequently if we examine a head and shoulders pattern on a small time scale there are five reversals, namely three peaks and two troughs. On this small time scale one does not have a complex pattern, but simply a reversal with no change in sign in the second derivative of price. Consequently, one might hope to use the candlestick charts to locate the right shoulder, for example, near the conclusion of a head and shoulders pattern formation.

We define a representative selection of candlestick patterns, and discuss how our approach would produce the particular pattern and provide an explanation for the dynamics of price movements. We refer to "long white body," etc. with the understanding that a precise statistical definition would be formulated based on the body length of recent history. Also, we assume some time elapses between any two trading days in terms of the differential equations due to the short time scale and the importance of a day's opening price relative to the prior day's close.

Morning Star, Morning Doji Star and Doji Star: These are all strong indications (no confirmation required) of a reversal from a downtrend into an uptrend (Figure 24). For the morning star the precise requirements, aside from being in a downtrend are:

i) The first day must coincide with the trend; i.e. black.
ii) The second day (the star) has a "body gap" from the first day and may have either color. That is, the open and close of the second day are strictly below the first day's close.
iii) The third day is white and has an upward "body gap" from the second.
iv) The first and third days are both longer than the second day.

If the second day is a doji (open and close price identical) then this is called a morning doji star. The definitions of evening star and evening doji star are the analogous reversals from uptrend to downtrend. The doji star refers to the first two days of either a morning or evening doji star.

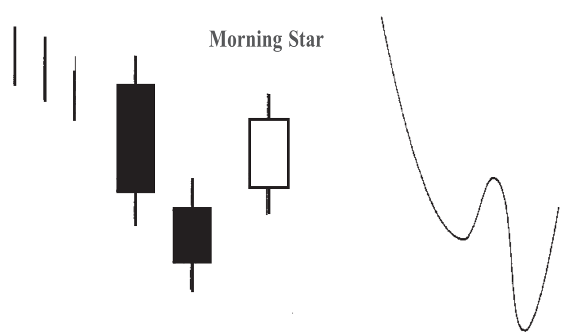


FIGURE 24: THE MORNING STAR: A reversal pattern occurs in a downtrend as a long black body is followed by a shorter body and then white body that closes well into the first day's body. Gaps occur between the consecutive days' bodies. The *morning star* is a decisive reversal sign that can be explained from the perspective of our theory quite naturally in terms of oscillations about a valuation function $P_a(t)$ that is U-shaped.

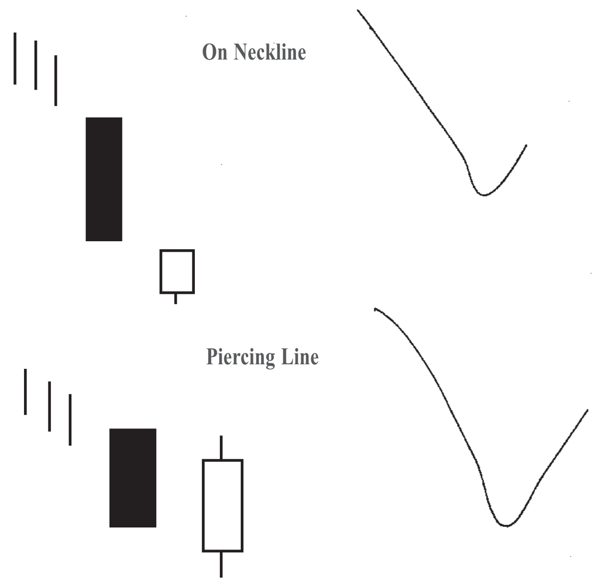


FIGURE 25: PIERCING LINE AND ON NECK LINE: Two patterns that appear similar actually have opposite implications are the *piercing line* and the *on neck line*. In both cases a downtrend continues as a long body is formed on the first day but a white body occurs on the second day. If this second day gaps lower but closes at the first day's low, then it is considered a bearish continuation pattern. A continuous approximation would indicate oscillations about a decreasing line as shown at the right. If the second day's rebound exceeds the midpoint of the first day's body, then the pattern is considered a reversal pattern, so that a 'buy signal' is given. From the perspective of our theory, the distinction between these two patterns is evident, since the larger rebound represents an oscillation (as shown on the right) that is incompatible with a decreasing valuation function $P_a(t)$.

From the perspective of the theory, each of these patterns represents a small oscillation about a linear $P_a(t)$, for example, as shown in Figure 24. Hence, in the head and shoulders pattern [Figure 14] any of the five maxima and minima would be similar to the continuous analog of the morning or evening star patterns.

Piercing Line and On Neckline: A pattern which appears to be quite different but is again similar to an oscillation about a constant $P_a(t)$ is the piercing line (Figure 25). The definition is specified by a downtrend followed by two days consisting of

i) A long black body for the first day.

ii) A white body for the second day which opens below the low of the previous day and closes above the midpoint of the first day's body.

The opposite pattern for a bearish reversal is called a dark cloud cover. Various modifications of these occur depending on the extent of the second day's rebound. If all features are the same as above except that the close of the second day is at the low of the first day then one has an "on neckline" which is a continuation pattern rather than a reversal.

Distinguishing these two patterns, as well as other related patterns that are in between in terms of the rebound (e.g. above the close but below the midpoint of the body), is accomplished most simply by assuming a (locally) linear $P_a(t)$ and examining the oscillation in P(t) as a function of the slope of $P_a(t)$. If the slope is zero near the relevant time, for example, then an initial overvaluation will result in overshooting $P_a(t)$ followed by a rebound. On the other hand, a rebound that does not go far into the previous day's is easily produced with either a negative slope or with an increased supply of stock submitted for sale. Figure 25 shows the differences in the extent of the rebound for a declining $P_a(t)$ versus one which has leveled off. Thus the extent of the rebound provides an indication of the perceptions of underlying value.

Other patterns which are related include "kicking" in which a black body in a downtrend is followed by a white body gapping upward, while both have a range defined by the body — i.e. no shadows (called "marubazu"). This is similar to the piercing line except in the abruptness of the transition. In terms of the model, the same values of q_1 and q_2 that produced a solution path with constant $P_a(t)$ will produce one that is consistent with kicking only if $P_a(t)$ itself exhibits a strong reversal. This is compatible with the designation of kicking as a strong reversal sign which requires no confirmation.

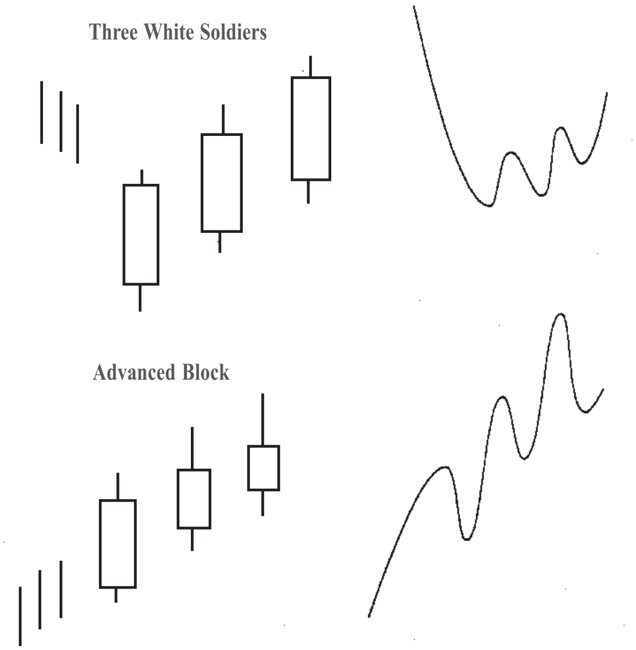


FIGURE 26: THREE WHITE SOLDIERS AND ADVANCE BLOCK: Following a downtrend, three white bodies appear consecutively as a reversal pattern, known as the *three white soldiers* forms. The continuum version of this is the formation of oscillations near a bottom leading to an uptrend. While the *advance block* formation is a similar three day pattern, it occurs in an uptrend and is characterized by a set of rules that imply a gradual 'loss of steam' indicating that the rally is petering out. From the continuum perspective, this pattern is a set of oscillations that is compatible with a peaking $P_a(t)$.

Three White Soldiers and Advance Block: The three white soldiers pattern (Figure 26) occurs in a downtrend and is defined by three days of trading that are all white bodies in which the open and close enclose each day's trading. The second and third days both open within the prior day's body. This defines "three white soldiers." A pattern similar to this is generated by *V*-shaped $P_a(t)$. The overlapping bodies are essentially oscillations about an increasing $P_a(t)$. The temporal length of the transition provides very strong confirmation for this reversal pattern.

A pattern that appears to be superficially similar, but is in fact a strong reversal sign, is the "advance block" (see Figure 26), which occurs during an uptrend and is defined by the following

i) All three days have white bodies with each of the last two opens within the prior day's body.

ii) The last two bodies are smaller than the first.

iii) The second day has an upper shadow which is much longer than the first day.

iv) The closing prices of all three days are all near one another.

While the stock price moves up during each day the rally is actually losing steam as the new closing highs are not significantly higher than the previous, and consequently it is a reversal signal. From the perspective of the theory, the

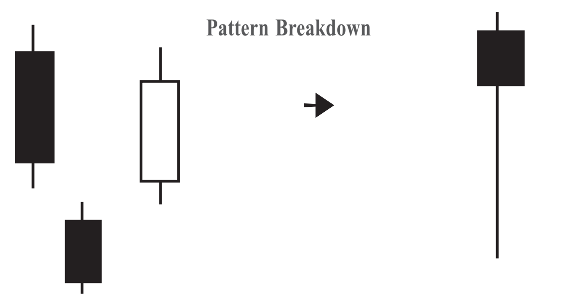


FIGURE 27: PATTERN BREAKDOWN: A change in time scale changes the appearance of the candlestick pattern. In this case, the three day *morningstar pattern* collapses into a single day *hammer pattern* upon rescaling time by a factor of three. The *hammer pattern* is characterized by a short body and a long shadow. The evolution characterizing the *morningstar* takes place in a single high volume day.

advance block can be modeled using a $P_a(t)$ that is first increasing, then decreasing. The opening prices that occur within the prior day's body provide the oscillations that resemble a small head and shoulders pattern. Hence, the model provides a simple explanation that distinguishes the bearish advance block from the bullish three white soldiers. The analogy of three white soldiers for an uptrend to downtrend reversal is called three black crows.

In some cases the patterns discussed can occur in one day. For example, the morning star collapsed into a single day results in a hammer pattern which has a short body and a long lower shadow, and a long downward shadow (Figure 27). Hence, the morning star that is reduced to a hammer is described by the same equations upon rescaling time.

Mat Hold: A five day pattern that occurs as a continuation pattern in an uptrend (Figure 28) is defined by the following:

1. A long white candlestick that establishes a closing high is formed in an uptrending market.
2. The second day has a small black body with a body gap from the first.
3. The second and third days also have small bodies (compared to the first) and have successively lower closes.
4. The fifth day is another long white day that closes at a new high.

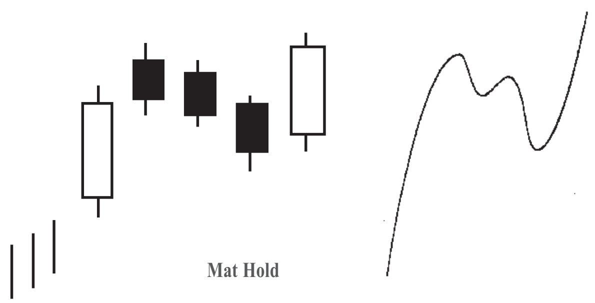


FIGURE 28: MAT HOLD: A five day pattern that occurs as a continuation pattern in an uptrend, called a *mat hold*, is characterized by three small bodies between two long white bodies. The fifth day closes at a high for the five days. The continuum analog of this occurs as two levels of oscillations in a sharp uptrend.

A less bullish version of mat hold is the rising three methods, in which the fifth day does not establish a new high. Both of these patterns are a simple consequence of the theoretical model in which $P_a(t)$ is a straight line with a large positive slope in the case of a mat hold and a small positive slope for the rising three methods.

The other continuation patterns can be similarly described within the theory. The central idea is that the candlestick patterns that are approximated by continuous functions are part of an oscillation about a monotonic $P_a(t)$ (that can be represented by a linear function due to the very short time scale). Many of these patterns are two day patterns and require confirmation as they form only part of an oscillation. The mat hold, which is very decisive, is a full cycle (i.e. comparable to $\sin x$ on $[0, 2\pi]$) and leaves no doubt about the underlying $P_a(t)$, unlike many of the two day patterns that form only a fraction of the cycle. Similar comments apply to reversal patterns.

6. Discussion and Conclusions

Our approach provides a coherent explanation of not only the patterns of technical analysis but also of the limitations involved in their application. One of the difficulties that afflicts technical analysis is that the rules and conclusions seem somewhat arbitrary and consequently unscientific. From the perspective of the asset flow model the validity of technical analysis is contingent upon only two factors: a finite asset base and the influence of trend based investing. The vast majority of our conclusions would not be altered with a somewhat different approach (e.g. discrete difference equations or a different form for the flow rate k) that nevertheless preserves these properties. This (or any similar) methodology elevates technical analysis from a set of arbitrary assumptions and conclusions to a set of implications that are implied by the unique solution of the differential equations. Given a set of parameters (trend based coefficient, etc.) that describe an investor population, one can obtain the implications of a particular configuration of price development as the unique solution of the equations.

In view of the simplicity of assumptions and the limited strategy involved in the model, it is somewhat surprising that these patterns emerge so naturally. In the symmetric triangle pattern, for example, one can imagine a complex set of strategies evolving in time as each local maximum or minimum is attained. The numerical results, however, seem to suggest the information included in the price trend and the extent of under/over-valuation is a suitable representation of the average of a wide spectrum of strategies.

There are two additional research problems that arise from this analysis. First, can patterns be detected in a market through statistical and computer testing, and if so, do they have predictive value? One would need to define the patterns in an algorithmic way that corresponds to human experts, as done in Kamijo and Tanigawa (1993) for some patterns. The next step would be the precise identification of the rules for deciding the trading action. Of course, a related issue is the possibility that these patterns can be created in a laboratory, perhaps using experienced participants. A second issue is the problem of deducing fundamental values, say $P_a^{(1)}$ and $P_a^{(2)}$, and assets of each group, given P(t). This is an inverse problem similar to inverse scattering.

Resolution of Some Nonclassical Phenomena

We return to some of the issues raised in the Introduction with respect to market phenomena exhibiting nonclassical behavior. By augmenting the classical price theory with the concepts of trend based investing and the flow of finite assets, one can explain a broad spectrum of phenomena.

i) Consider a worthless stock which begins to rally as unknowledgeable investors buy due to takeover rumors that they (mistakenly) believe will raise the price to the stock which currently trades for $1. More informed investors may sell this stock short in the expectation that the stock will eventually attain its true value of $0. However an investor who has a total of $1000 in his brokerage account and sells short 1000 shares is aware that he must begin purchasing back shares if the price moves above $2. In fact, the brokerage will do this unilaterally if he does not (if no additional cash is brought into the account). If the funds of the unknowledgeable investors exceed those of the short sellers and others selling into the rally, then the price will move up from $1. The investor who sold short at $1 must be very concerned with the trend since a focus solely on fundamental value (of $0) will quite possibly lead to a loss. Furthermore, if the price does move sufficiently higher then the short seller is motivated to close his position by purchasing the shares and thereby contributing to further lifting of the price. Even in this very obvious valuation, the knowledgeable investor is motivated to have a preference that depends on a price derivative or trend that he knows expresses the preferences of unknowledgeable investors. Hence one may have a bubble (as in the laboratory experiments) and a subsequent crash which may be prompted due to various reasons. One of these would be a published report that the stock is worthless, thereby eliminating almost all of the buyers, and changing the price derivative that induces others to sell solely for that reason. Another, but slower mechanism, is that eventually the supply of cash of the uninformed investors is exhausted.

ii) The large bubbles that have occurred during a government's sale of stock (e.g. NTT) in a privatization program seem paradoxical even in view of the fact that the government is increasing supply. However, the huge amount of free advertising that is available to a government tends to increase the demand among the public, often in excess of supply. A series of offerings in which the initial subscribers have always seen immediate (paper) profits that are widely reported tend to bring a large demand at the next such sale.

As in the other examples the resources of the new entrants to a particular financial instrument are ultimately overwhelmed by those of the more established group that is more concerned with economic fundamentals, and prices fall. In summary, the bubbles that arise during highly publicized government offerings are initiated by a dramatic enlargement of demand, which creates a trend, followed by further rising prices due to trend based investing. The implications for efficient market theories and the trend based asset flow are quite different in terms of public policy. While the efficient market hypothesis would maintain that prices are at fair value, the asset flow analysis indicates that careful consideration must be given to balance the supply of new stock with the new demand created by publicity, particularly since a similar amount of publicity will be created by the bubble's demise, thereby choking off additional demand for other new issues.

iii) Many of the world's bubbles appear to have an origin that is modeled in the upside breakout from a rising channel (e.g. Jakarta in Figure 9) as follows. A rising trend develops in response to steadily rising corporate earnings as the dominance by investors focused on economic fundamentals maintains the growth of stock prices to value, e.g. at 20% per year. The steady and seemingly certain increase of share prices attracts the attention of a new and less knowledgeable group of investors who bring a fresh supply of cash, a vague notion of value and a strong attachment to the trend. The value based investors are also aware of the trend, and as in the Mexico example, do not seem inclined to sell when a powerful rally has begun. However, once the new group begins to run out of funds, the rally begins to falter and the 'buying is exhausted.' The value based investors sell out more quickly simply because the value part of their preference is heavily negative at the height of the bubble while the trend part is just mildly positive as the rally begins to peter out, hence the sum is negative. On the other hand the new investors have a positive preference at, and slightly *after*, the peak. Once the trend has clearly reversed, the preference of both groups is negative and the rout cannot end until the price is at or below fundamental value and the value based investors resume buying.

iv) The large declines or crashes in the US and other markets in 1929 and 1987 have been discussed in Caginalp and Balenovich (1991), (1994b). Using a constant value for $P_a(t)$ (since economic fundamentals are relatively unchanged in one week), and the same parameters that were calibrated from the experiments, one obtains good agreement with the time evolution for the first few days. In particular the results explain the large rebound that falls in between the top and bottom prices, and is difficult to explain from an equilibrium perspective. A similar analysis has been applied to the large premiums that evolved on the country closed-end funds, (v). In both 1929 and 1987 the longer term behavior may be described as reaching an overvalued state before being confronted by a full percentage discount rate hike that effectively lowered the value of stocks at an overextended time.

v) The persistence of discounts in the average closed-end fund has an immediate explanation from our perspective. The funds are initially priced about 6 to 10% above net asset value (NAV) in order to cover costs. Also, the attempts to sell to a broad base of investors before the offering may lead to a smaller group of potential buyers after the offering. There is also the possibility that the underwriters are left with some stock that must be sold shortly *after* the offering. The price is essentially determined but the underwriters (e.g. short selling at the outset is generally disallowed) so that a premium can be imposed. However, in the absence of a stellar performance of the NAV the value base buying and selling short will initiate a downward trend. The trend based investing will then result in crossing below the NAV and remaining below it.

A sample evolution is shown in Figure 29, where $P_a(t)$ is \$10.50. The initial price is \$10.50, while B(0)=0.56 so that more than half of the potential market is saturated. The other parameters are $q_1 = 175$, $c_1 = .0005$, $q_2 = 1$, $c_2 = .001$. The stock falls soon after opening and reaches a steady state of only \$9.30 which is an 11.4% discount, even without an initial overvaluation. In fact if one simply analyzes the dynamical equilibrium of equations (3.4) - (3.6), (3.9) -(3.11) it is clear that one must have k = B and $\zeta_2 = q_2 (1 - P / P_a(t))$ in order to have constant price as t approaches infinity. Since $k = (1 + \zeta_2) / 2$ a discount in

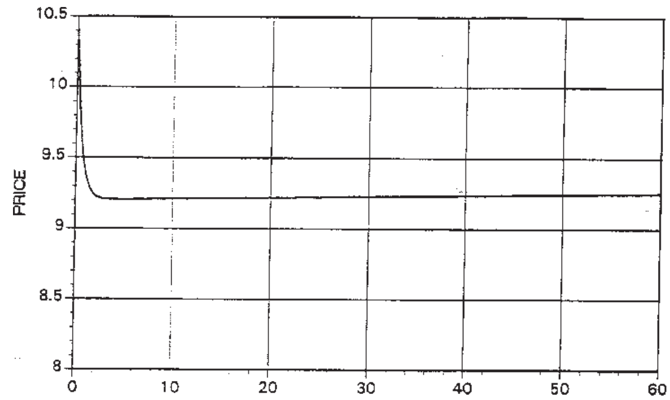


FIGURE 29: THE CLOSED END FUND PUZZLE: The dependence of the equilibrium price on the initial price and fraction of investors holding stock is a possible explanation of the persistent discount that afflicts closed end funds. Here we start with 56% of the investment capital invested in the stock. The $P_a(t)$ is constant at \$10.50. Even in the absence of initial overvaluation, the steady state price that emerges is about \$9.30. An initial overvaluation results in a lower drop in the intermediate stage.

price is compatible with $B > 0.5$. Thus the equilibrium price, which depends upon initial conditions and parameters, will be lower than the NAV because of the initial overvaluation and an 'excess' fraction of investors already invested in the fund. The remedy to this problem would be an industry wide shift toward paying for costs over a long time period rather than through the initial offering, and to sell less aggressively *before* the offering and continue marketing steadily after the opening.

In summary, a broad range of phenomena and puzzles observed by practitioners can be explained by the asset flow theory that is a generalization of the classical theory of adjustment augmented by the concepts of finite assets and trend based decisions (in addition to value based). In particular, technical analysis patterns can be explained largely on the basis of oscillations about a changing assessment of fundamental value, or on the complex dynamics generated by multiple groups with varying assessments of value. In the latter, a key issue involves the relative assets of the two groups, so that a breakout occurs as one group's resources (cash or stock) is sufficiently diminished.

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Footnotes

1. *The paper is a revision of a University of Pittsburgh Preprint (1994a) with the same title.*
2. *Lo et al. [2000] also adopted a similar first sentence, and similarly argue that the two groups are difficult to reconcile due to the difference in motivations as well as language.*

Biographies

Dr. Gunduz Caginalp is a Professor of Mathematics at the University of Pittsburgh. He received his PhD in Applied Mathematics from Cornell University in 1978 after obtaining bachelor and masters degrees there. He has also held academic positions at The Rockefeller University and Carnegie-Mellon University. He is the Editor of the Journal of Behavioral Finance, a member of the Editorial Board of Applied Mathematical Finance, and author of over 80 journal articles.

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